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A SINUSOIDAL-VIBRATION ANALYSIS PROGRAM FOR EXPERIMENTAL DATA

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16. Abstract

A program is described for obtaining resonant frequencies, modal amplitudes, and damping of a structure from a sinusoidal force-controlled vibration test. Presented are the theoretical basis for the analysis (the Kennedy-Pancu method), recommendations for a test procedure based on experience with several earlier test programs, and an outline of the data analysis technique. Although the program was developed for structural vibration problems, variations for processing any sinusoidal data are available.

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SUMMARY

A program is described for obtaining resonant frequencies, modal amplitudes, and damping of a structure from a sinusoidal force-controlled vibration test. The theoretical basis for the analysis, recommendations for test procedures, and description of the data analysis technique are also presented. As a demonstration of the use of these techniques, a sample problem is included in the appendix. Although the program was developed for structural vibration problems, variations for processing any sinusoidal data are available.

INTRODUCTION

The primary purpose of a resonant vibration test generally is to determine the natural frequencies, principal mode shapes, and the damping of a given structure. Through the years many methods have been developed to identify and evaluate these parameters. Bishop and Gladwell (ref. 1) present a theoretical background to the problem of resonant testing and then discuss various techniques in relation to this theory. Vibration test techniques fall into one of two categories: (1) several exciters placed along the structure applying forces in the correct level and phase relationship so as to excite the principal mode at the appropriate natural frequency and (2) a single exciter applying a point force to the structure at some arbitrary location. Usually the multiple-exciter method requires little data processing but is very complicated to set up and requires much equipment. The single-exciter method is much simpler to set up; however, the response data must be processed if correct information on the frequencies, modes, and damping is to be obtained. Pendered (ref. 2) states that because of the expense and complexity of the experimental procedure using multiple shakers, most investigators must place their reliance on the simpler test technique of using a single-point excitation source. When the single-point excitation source is used, several methods of data interpretation are available, such as the peak amplitude method, the quadrature response method, or the maximum quadrature component method. Although the simplicity of these methods is attractive, both Bishop and Gladwell (ref. 1) and Pendered (ref. 3) recommend the more complicated method of Kennedy and Pancu (ref. 4) as the best method of data interpretation presently available.

Three advantages of this method, as outlined by Bishop and Gladwell, are (1) "its ability to show up the existence of modes," (2) "it provides a simple way of determining with fairly good accuracy the true peak amplitude in the resonant mode at a resonant frequency," and (3) "by giving a means of extracting the vibration in the resonant mode from the total vibration, it allows a better estimate to be made of the damping coefficient."

The Kennedy-Pancu method has been successfully used in the investigation described in references 5 and 6, and on the basis of the experiences of these programs a technique for testing and analyzing vibratory data has evolved. The purpose of this report is to present this technique which includes recommendations for test procedures and a guide to a recently developed computer program for sinusoidal data analysis. In the computer program, experimental data, recorded on magnetic tape as part of a structural vibration test using a single-point force-controlled excitation source, are analyzed by employing the method of Kennedy and Pancu to obtain resonant frequencies and modes. Damping values are calculated by using the phase-change method outlined by Mead (ref. 7) on the reduced data. This program may be used either for steady-state data, that is, fixed frequency increments, or for a more convenient method of slowly sweeping through the frequency ranges of interest to locate the resonant frequencies.

This report outlines the method of Kennedy and Pancu for obtaining resonant frequencies and modes, presents the phase-change method of calculating damping, presents the data analysis processing methods, and outlines some available variations of the basic program. Included in the appendix are the requirements for obtaining experimental data and a sample case. To the experimenter interested in using this method of data analysis, it is recommended that the following sections of this report be read first in the stated order:

(1) Experimental Data Analysis Concept
(2) Basic Flow of Data Analysis Processing
(3) Appendix

To the programer interested in implementing this program, it is recommended that the following sections of this report be read first in the stated order:

(1) Data Analysis Method
(2) Appendix

SYMBOLS

A	amplitude at fundamental frequency of a signal, engineering units
A_{ha}	amplitude obtained from harmonic analysis equation, engineering units

A_n	normalized amplitude at fundamental frequency of a signal
A_r	amplitude at fundamental frequency of normalizing signal, engineering units
A_v	nondimensional amplitude at fundamental frequency of a signal
AC	alternating voltage signal representing a physical quantity
$a_0, a_1, \ldots, a_n$	Fourier coefficients of cosine term in harmonic analysis
B_l	average lower band edge for discriminator slopes, counts
B_u	average upper band edge for discriminator slopes, counts
$b_1, b_2, \ldots, b_n$	Fourier coefficients of sine term in harmonic analysis
C,D,E	coefficients of circle equation
c	viscous damping coefficient, N-sec/m
c_c	critical damping coefficient for a viscous system, N-sec/m
d	resonant diameter of circle
$F(t)$	forcing function, N
f	frequency, Hz
Δf	frequency difference between two consecutive points, Hz
f_{co}	filter cut-off frequency, Hz
f_i	frequency at x_i, y_i used in computing $\Delta s/2f\Delta f$, Hz
f_m	resonant frequency as determined by maximum $\Delta s/2f\Delta f$, Hz
f_1	lower frequency used in computing damping, Hz
f_2	upper frequency used in computing damping, Hz

G	gain setting for each channel
g_1, g_2	slopes of chord lines describing damping angle
IRIG	Inter-Range Instrumentation Group
$j = \sqrt{-1}$	
K_A	attenuation constant for amplitude correction
K_{AI}	intercept of attenuation curve at calculated value of f/f_{co}
K_{AS}	slope of attenuation curve at calculated value of f/f_{co}
K_D	discriminator slope, 1/count
K_S	sensitivity constant, engineering units
K_T	correction time for IRIG head tape offset, sec
k	stiffness, N/m
L	calibration level, engineering units
m	mass, kg
n	time duration of two cycles, sec
o_1, o_2	second sums in computing harmonic analysis
p	period, sec
$Q_0, Q_1, \ldots, Q_{11}$	amplitudes of 12 evenly spaced samples of data for each channel, engineering units
q_1, q_2	second differences in computing harmonic analysis
R	ratio of recorded speed to playback speed

r	radius of circle
s	arc length
Δs	distance between consecutive data points
t	time, sec
u_1, u_2, u_4, u_5	first sums in computing harmonic analysis
$V_0, V_1, \ldots, V_{11}$	nondimensional amplitudes of 12 evenly spaced samples of data for each channel
$v_0, v_1, \ldots, v_5$	first differences in computing harmonic analysis
W	cut-off frequency of low-pass filter in discriminators, Hz
X, Y	normalized coordinates along the real and imaginary axes
X_i, Y_i	normalized coordinates at point i
x, y	coordinates of a point
x_c, y_c	coordinates of center of a circle
x_d, y_d	coordinates of displaced origin
x_m, y_m	coordinates of f_m at maximum $\Delta s/2f\Delta f$
x_1, y_1	coordinates of f_1
x_2, y_2	coordinates of f_2
$Y_0, Y_1, \ldots, Y_{11}$	amplitudes of 12 evenly spaced samples of data for each channel, counts
$Z_0, Z_1, \ldots, Z_{11}$	amplitudes of 12 evenly spaced averaged samples of data for each channel, counts

$\beta = \omega/\omega_n$

θ	relative phase angle, deg
θ_{adj}	phase angle from harmonic analysis adjusted for interpretation purposes, deg
θ_{ha}	phase angle obtained from harmonic analysis equation, deg
θ_{irig}	phase-angle correction for IRIG head tape offset, deg
θ_m	correction factor equal to either 0° or 360°
θ_{min}	minimum value of θ_{adj}, deg
θ_{orient}	phase-angle orientation correction, 0° or 180°
θ_r	relative phase angle of reference channel, deg
μ	damping factor, $2c/c_c$
ϕ	referenced phase angle, deg
$\Delta\phi$	change in phase angle between f_2 and f_1, deg
ω	circular frequency, rad/sec
$\Delta\omega$	change in circular frequency, rad/sec
ω_n	natural frequency, rad/sec

Dots over symbols indicate time derivatives.

EXPERIMENTAL DATA ANALYSIS CONCEPT

Theoretical Method

The basis for this data interpretation program is the method of Kennedy and Pancu (ref. 4). This method has been shown to be a superior procedure in obtaining resonant frequencies when they are either very closely spaced (ref. 3) or when a peak in the response curve is not apparent (ref. 4). The analysis assumes that (1) the vibrating

structure is a linear elastic system with either hysteretic or viscous damping, (2) the "off-resonant" contributions in the immediate vicinity of resonance are constant in both amplitude and phase, and (3) the modes are not coupled by damping. In matrix mathematics, this last assumption means that the inertia, stiffness, and damping matrices can be diagonalized by the same linear transformation of the coordinate vector. The magnitude and phase, relative to an applied force, of an experimentally determined motion response vector are plotted in the complex plane. By use of the aforementioned assumptions, it is possible to investigate the locus of the tip of the total response vector and to divide it into a resonant response and an off-resonant response. The resonant response is then analyzed as a single-degree-of-freedom system.

To help understand this method, the following example of the response of a single-degree-of-freedom system is analyzed by using the Kennedy-Pancu approach. The system, as shown in figure 1, is represented by a mass connected to the ground by a linear spring and a viscous damper and is excited by a sinusoidal force applied to the mass. The solution of the equation is presented as a ratio of vibratory acceleration to applied force. This ratio is sometimes referred to as inertance. Although acceleration is not commonly used in analytical presentations, experimentally the availability of acceleration-sensing instruments operable over a wide frequency and magnitude range makes this measurement very convenient to obtain. Solving for the inertance at the input results in the following equation:

$$\frac{\ddot{x}}{F(t)} = \frac{1}{m - \frac{k}{\omega^2} - j\frac{c}{\omega}} \tag{1}$$

where

$\ddot{x}$ acceleration, m/sec^2

$F(t)$ forcing function, N

m mass, kg

k stiffness, N/m

c viscous damping coefficient, N-sec/m

ω circular frequency, rad/sec

$j = \sqrt{-1}$

Nondimensionalizing the parameters and separating the real and imaginary parts yields

$$\frac{m\ddot{x}}{F(t)} = \frac{\beta^4 - \beta^2}{\left(\beta^2 - 1\right)^2 + \mu^2\beta^2} + \frac{j\mu\beta^3}{\left(\beta^2 - 1\right)^2 + \mu^2\beta^2} \tag{2}$$

where

$$\beta = \frac{\omega}{\omega_n}$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$\mu = 2\,\frac{c}{c_c}$$

$$c_c = 2\sqrt{km}$$

The inertance leads the input force by the phase angle ϕ where

$$\phi = \tan^{-1}\frac{\beta\mu}{\beta^2 - 1} \tag{3}$$

A plot, in the complex plane, of the nondimensionalized inertance (eq. (2)), using $\mu = 0.020$, is shown in figure 2. The circular shape of the response curve is the starting point for the Kennedy-Pancu technique. The significant parameters of resonant frequency, modal amplitude, and damping are discussed next.

Resonant Frequency

The resonant frequencies are located by determining maximums in a curve of the change in arc length (along the path of total response) divided by the change in the squared frequency $\left(\text{i.e., } ds/d\left(\beta^2\right)\right)$ as a function of frequency. Each maximum is defined to be a resonant frequency. The parameter $ds/d\left(\beta^2\right)$ is equal to $ds/2\beta d\beta$. For the program described later in this report, by using small increments of frequency, the parameter $\Delta s/2f\Delta f$ is calculated to determine resonant frequencies (the parameter β is a frequency ratio term and in effect is fully equivalent to f).

Mode Shape

The amplitudes used to determine the mode shapes are obtained from a fitted circle. By using the assumption that the off-resonant contributions are constant around resonance, then the diameter of a circle fitted to the data near the resonant frequency represents the

modal, or resonant, component of the response. In the example presented (fig. 2), it may be observed that the diameter of the circle and the magnitude of the total inertance response are the same; however, in a more complex system, this is not always the situation. In the more complex case, the method of determining the modal component is the same, that is, fit the "best circle" through the data near resonance (maximum $\Delta s/2f\Delta f$). The point diametrically opposite the maximum $\Delta s/2f\Delta f$ is called the "displaced origin" for this mode. A vector from the origin of the coordinate system to the displaced origin represents the "off-resonant" vibration present in the total response. An example of such a situation may be seen in figures A15 and A18. The mode amplitudes are determined by dividing only the diameters of the circles by the diameters of a reference location.

Damping by Phase-Change Method

The modal damping value is calculated by using the phase-change method outlined by Mead (ref. 7). To use this method it is necessary that (1) the measurements be made on the "resonant peak," that is, in the region of frequency where the real component of response maintains a relatively constant slope on a response-frequency graph (the real component is zero in this region for a one-degree-of-freedom system), (2) the frequency of the exciting force be very stable and be varied continuously through the resonant peak (experience has shown that steady-state data usually are not sufficiently accurate, but good results have been obtained by slowly sweeping the frequency), and (3) the frequency increments measured during analysis of the sweep data be accurate. These particular requirements are very compatible with the complex plane method of interpretation. The parameters used to calculate damping are obtained by using the best circle representing the modal response as a single-degree-of-freedom system. The phase angles are then measured from the displaced origin. Rewriting equation (3) in the dimensional form gives

$$\tan \phi = \frac{c\omega}{m\omega^2 - k} \tag{4}$$

Differentiating this equation and evaluating the results in the immediate range of the resonant frequency results in

$$\frac{\Delta\phi}{57.2958\Delta\omega} = \frac{2}{\omega_n \mu} \tag{5}$$

or

$$\mu = \frac{114.5916\Delta\omega}{\omega_n \Delta\phi} = \frac{(114.5916)(f_2 - f_1)}{f_m \Delta\phi} \tag{6}$$

where

$\Delta\phi$ change in phase angle between f_2 and f_1, measured from best circle as shown in figure 2, deg

f_m resonant frequency, Hz

f_1 lower frequency used in calculating damping, Hz

f_2 upper frequency used in calculating damping, Hz

The procedure just outlined is for a viscous damped system; however, the interpretation would be similar if the damping were for a hysteretic system (refs. 1, 7, and 8). By using the example shown in figure 2, equation (6) yields a damping factor of 0.02, which agrees with the originally selected value of damping.

DATA ANALYSIS METHOD

A program has been developed to analyze structural vibration data. The basis of this program is the Kennedy-Pancu technique described in the preceding section. The program is designed to analyze data obtained from a single-shaker force-controlled vibration test and provide the modal amplitudes, resonant frequencies, and damping of the test structure. Since the method of Kennedy and Pancu is a procedure for improving the interpretation of experimental data, this program also provides information to aid in evaluating the accuracy of the aforementioned parameters.

In the next several sections are outlined the procedures which have been successfully implemented in obtaining and processing experimental data by using the Kennedy-Pancu method. In the first section, a description is given of the path the data follow while being converted to the proper format for analysis. In the next section, an outline is presented of the various steps in which the data are analyzed. It is important that an experimenter, using this technique, understand this section. In the last section, some of the details used to incorporate these procedures into the program are described.

ADTRAN Processing

The analog-to-digital transcriber (ADTRAN) processing system is designed to interpret data recorded on a 14-channel analog tape recorder. The analog tapes containing calibration and experimental data must be converted to a digital format. To permit the digitizing of the data by the ADTRAN system, the tape recorder must be preset and operated according to the information given in table I. Listed in this table are the frequency

limits of the transcriber used to develop this program. It should be noted that for this particular system, these limits are considerably more restrictive than those limits of an FM tape recorder. A different transcriber system would have different limits. The flow of data through a central data transcription is shown in figure 3. The FM tapes are played through a set of discriminators which convert the FM signal to the analog signals originally recorded at the test facility. These signals are then sent to the analog input section of the analog-to-digital transcriber, which stores all the analog voltages simultaneously. The stored analog voltage for each channel is converted to a digital number in the transcriber, and the digital numbers are then put in proper format for recording on a digital computer tape.

The data from the time code generator, recorded on channel 14, are fed into a time code translator, which decodes the time code information and formats the data for proper entry into the transcriber. If a modified NASA 36-bit time code generator is used, this information generally includes a run number, a data group number, and elapsed time.

The digitized analog test data, time code information, and additional digital constants to further identify the data are put in proper format and recorded on digital magnetic tape for computer entry.

To control the samplings of the data at the correct time, some additional control equipment is used to detect the zero crossing of a reference signal. This signal, generally recorded on channel 13, must have a frequency identical to that of the forcing function and must be maintained at a constant amplitude with minimum noise and distortion. An oscillator, which automatically phase locks to the reference signal, runs at exactly 12 times the reference frequency and is used to generate the 12 sampling intervals per cycle. The control equipment also generates a sample number and cycle number to identify each sample of data; these numbers are recorded with each sample of data. The cycle number, sample number, 12 channels of data, and reference signal are patched for an ADTRAN digitizing board as shown in figure 4.

Shown in figure 5 is the format used to record the information just described on the computer tape. This digitized tape is written in binary coded decimal with 333 words per tape record. The first nine words of the record are identification numbers and constants used to reduce the data to engineering units. The rest of the words are divided into groups of nine words, each containing 12 channels of data. The first two words of this group are for identification. The next six words have 12 analog channels packed two channels per computer word. The last word in this subgroup is a dummy. This grouping allows the record to hold 12 sample values for each of 3 cycles of data per channel.

Each digitized tape may have three types of records: (1) discriminator records, (2) constant-frequency calibration records, and (3) data records. The second and third

types of records are self-explanatory. The purpose of the discriminator records is as follows:

(1) Discriminator slopes are computed for each analog channel. The discriminator slope is used to remove the effects of day-to-day variations of the discriminator on the data. Five sets of counts (a digital level of measurement) are recorded at each of three levels: lower band edge B_l, zero, and upper band edge B_u. An average value is computed at each of the three levels, and the computed average is compared with each sample. Any sample which is not within ±10 counts of the average is discarded, and a new average is computed. This process is repeated until all remaining samples are within ±10 counts of the average. There must be at least three samples remaining or discriminator slopes are not computed. The discriminator slope K_D is computed from

$$K_D = \frac{2000}{B_u - B_l} \tag{7}$$

where $B_u - B_l$ is the range of the discriminator in counts and 2000 is the arbitrary constant used in computing discriminator slope.

(2) The speed ratio R which is the ratio of recorded speed to playback speed, is logged in word 1 of the discriminator records. The speed ratios that can be used in digitizing the data are 64, 32, 16, 8, 4, 2, 1, 1/2, 1/4, 1/8, 1/16, 1/32, and 1/64. The speed ratio information is shown in table I.

(3) Electronic filters are used in the analog-to-digital transcription equipment in digitizing the FM signals. The cut-off frequency of the low-pass filter in the discriminators W is logged into word 2 of the discriminator records.

Basic Flow of Data Analysis Processing

The process for obtaining resonant frequency, mode shape, and damping from the sinusoidal data recorded on 14-channel analog tape is presented in 10 steps. It is necessary that the experimenter make a decision at the end of some steps to allow the program to continue. Some steps may be omitted if desired. The procedure is outlined as follows:

(1) AC calibrations – This run obtains amplitude, relative phase angle, and frequency information necessary to calculate the instrument sensitivities and the phase corrections due to tape IRIG head offset for each channel.

(2) Data averaged and converted to engineering units – In the same manner as the calibrations, the data are digitized for all 12 channels by using 12 samples per cycle for each 9 cycles over the frequency range being analyzed. The 9 cycles are averaged and converted to engineering units and the frequency is calculated from the period. This averaged value now represents a single data point in the frequency range of interest.

(3) Amplitude and phase-angle calculations – The amplitude and phase angle of the fundamental frequency of the signal are computed from a 12-point harmonic analysis by using a Fourier series. The amplitude is corrected for digitizing effects and the phase is adjusted for interpretation, corrected for offset in the IRIG head, and corrected for transducer orientation.

(4) Digitally filtered frequency, amplitude, and relative phase angle – Because digital calculations may cause scatter in the data, digital filtering is used for smoothing the frequency, amplitude, and relative phase angle.

(5) Filtered normalized amplitude, referenced phase angle, and calculated $\Delta s/2f\Delta f$ – The amplitude of each channel is normalized by the amplitude of a reference channel. The relative phase angle of a reference channel is subtracted from the relative phase angle of each channel to obtain referenced phase angle. The parameter $\Delta s/2f\Delta f$ is calculated. These data are stored on magnetic tape and are now ready to be plotted according to the selected method.

(6) Frequency response plots – The standard plot option consists of the logarithm of the $\Delta s/2f\Delta f$ function (DS/2FDF), the referenced phase angle (NPA), and the logarithm of the normalized amplitude (NA) as a function of frequency (FREQUENCY, HZ) for each channel. For reference purposes, markers showing intervals of 100 data points are also plotted on the frequency scale. An example of this plot is shown in figure A11. Note: <u>These plots must be reviewed by the experimenter and relatively narrow frequency ranges of interest must be determined before either of the next two steps can begin.</u> To aid in making this decision, the experimenter should read steps (7) and (8).

(7) Polar plots – The normalized amplitude is plotted as a function of the referenced phase angle for a given frequency range in polar coordinates. These plots may be omitted if the experimenter can determine by other means the exact frequency ranges to be evaluated for each mode. Otherwise, these plots act as a guide in determining these frequencies. Five frequencies (the lowest, three intermediate, and the highest in each range) are labeled on the polar plot. A maximum of 500 data points may be plotted. An example of this plot is shown in figure A13. Note: <u>These plots must be reviewed by the experimenter if this option is selected and relatively narrow frequency ranges must be determined before the circle curves are fitted to the data.</u>

(8) Calculation of resonant frequency and damping – By using the frequency ranges specified in either step (6) or step (7), the resonant frequency can be calculated. A maximum of 200 data points may be used in these calculations. The procedure is as follows:

(a) Fit circle to data – A least-squares method is used to fit a circle to the data.

(b) Force data points to circle – The data points are forced to the circle along the radius line from the center of the circle to the measured data point (if necessary). These adjusted coordinates are used in calculating the damping.

(c) Calculate resonant frequency and displaced origin – A maximum value of $\Delta s/2f\Delta f$ in each limited frequency range is determined and the frequency at that point is identified as a resonant frequency. A point on the fitted circle diametrically opposite the maximum value of $\Delta s/2f\Delta f$ is the displaced origin.

(d) Calculate damping – Damping is calculated from the circle data by using equation (6). The resonant peak is assumed to be included in a range of $\pm 10^{\circ}$ from the displaced origin about resonance and the frequencies at these angles are used in the equation. If there are insufficient data, either a 10° or a -10° span is used and is marked with an asterisk on data tabulation.

(9) Circle-fitted resonant response data plots – The circle-fitted resonant response data plots are an option. These plots may not be necessary; however, they do show the results of the calculations of step (8) in graphic form. The normalized amplitude is plotted as a function of the referenced phase angle in polar coordinates. The fitted circle, the resonant diameter drawn through the displaced origin, and the phase angle used in calculating damping are included in the plot. In addition, a plot of $\Delta s/2f\Delta f$ as a function of frequency is included in the same figure. Examples of this type of plot are shown in figures A15 and A18.

(10) Mode plots and tabulated listing of resonant frequency and damping – Final results of this program are as follows:

(a) Mode plots – The mode shape plot is a plot of the normalized diameter of the fitted circle as a function of the normalized station for each channel. Examples of this type of plot are shown in figures A16 and A19.

(b) Tabulated listing of resonant frequency and damping – The test identification, the given beginning and end frequencies of the mode search, and the frequency increment are indicated at the top of the listing. The channel number, normalized station, resonant frequency, resonant diameter, and damping are listed in columns for each channel. Examples of this listing are given in figures A17 and A20.

Calculation Procedures

Some of the procedures used to implement this program are presented in the same order as in the section entitled "Basic Flow of Data Analysis Processing."

(1) AC calibrations – Information obtained from the calibration signals is as follows:

(a) Frequency – The time duration of 2 cycles of data n is determined from the reference signal while the data are being digitized. The frequency f for each 9-cycle data point is then calculated from

$$f = \frac{R}{n/2} \tag{8}$$

(b) Amplitude and relative phase angle of the fundamental frequency – The amplitude and relative phase angle of the fundamental frequency are calculated for each channel by using a 12-point-per-cycle sample for 9 cycles. The symbols $Y_0, Y_1, \ldots, Y_{11}$ represent the amplitudes of 12 evenly spaced samples for 1 cycle of data in counts. The symbols $Z_0, Z_1, \ldots, Z_{11}$ represent the amplitudes of the average of 12 samples of data for 9 cycles where the cycles are counted from 0 to 8 and are shown as superscripts. For example,

$$Z_0 = \frac{Y_0^0+Y_0^1+Y_0^2+Y_0^3+Y_0^4+Y_0^5+Y_0^6+Y_0^7+Y_0^8}{9} \tag{9}$$

The amplitudes Z_1 through Z_{11} are obtained in the same manner. The 12 averaged samples are nondimensionalized by

$$V_0 = Z_0 K_D \tag{10}$$

where

V_0 nondimensionalized amplitude of the first sample of data

The nondimensional amplitudes V_1 through V_{11} are obtained in like manner. A harmonic analysis using a Fourier series (ref. 9) is performed on the averaged measured response data for 12 ordinates. The basis for the harmonic analysis is that any periodic function can be presented by a trigonometric series of the form

$$y = a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \ldots + a_n \cos n\omega t$$

$$+ b_1 \sin \omega t + b_2 \sin 2\omega t + \ldots + b_n \sin n\omega t \tag{11}$$

By knowing equidistant values of the independent variable of the function, the unknown constants $a_0, a_1, \ldots, a_n, b_1, b_2, \ldots, b_n$ are determined. The amplitude of the fundamental frequency and the corresponding phase angle are calculated by using equations (12) to (15) and the average amplitude of the 12 samples, as follows:

$$a_1 = \frac{1}{6}\left(v_0 + \frac{\sqrt{3}}{2} q_1 + \frac{1}{2} q_2\right) \tag{12}$$

$$b_1 = \frac{1}{6}\left(v_3 + \frac{1}{2} o_1 + \frac{\sqrt{3}}{2} o_2\right) \tag{13}$$

where

$$u_1 = V_1 + V_{11}$$

$$u_2 = V_2 + V_{10}$$

$$u_4 = V_4 + V_8$$

$$u_5 = V_5 + V_7$$

$$v_0 = V_0 - V_6$$

$$v_1 = V_1 - V_{11}$$

$$v_2 = V_2 - V_{10}$$

$$v_3 = V_3 - V_9$$

$$v_4 = V_4 - V_8$$

$$v_5 = V_5 - V_7$$

$$q_1 = u_1 - u_5$$

$$q_2 = u_2 - u_4$$

$$o_1 = v_1 + v_5$$

$$o_2 = v_2 + v_4$$

The amplitude obtained from the harmonic analysis equation is

$$A_{ha} = \sqrt{a_1^2 + b_1^2} \tag{14}$$

The phase angle is obtained from

$$\theta_{ha} = \tan^{-1} \frac{b_1}{a_1} \qquad (-180^\circ \text{ to } 180^\circ) \tag{15}$$

The electronic filter used in the analog-to-digital transcription equipment attenuates the signal and, as a result, an amplitude correction factor must be applied to the digital data. There is no need for a phase correction factor. The amplitude transfer function for the filter (fig. 6) is normalized by a filter cut-off frequency f_{co}. To determine the frequency cut-off, the following formula is used:

$$f_{co} = RW \tag{16}$$

The filter attenuation constant K_A is obtained from

$$K_A = \frac{f}{f_{co}} K_{AS} + K_{AI} \tag{17}$$

where

K_{AS} slope of the attenuation curve at calculated value of f/f_{co}

K_{AI} intercept of the attenuation curve at calculated value of f/f_{co}

The nondimensional amplitude of the fundamental frequency of a signal A_v corrected for filter attenuation is

$$A_v = A_{ha} K_A \tag{18}$$

(c) Sensitivity – The sensitivity constant K_S in engineering units for each channel of data is calculated from

$$K_S = \frac{L}{A_v} \tag{19}$$

where

L calibration level, engineering units (supplied on the form shown in fig. A4)

(d) Phase-angle adjustment and IRIG offset time-correction-factor calculations – The output of the program gives the relative phase angle in terms of the angle at which the test signal leads the reference signal. To be consistent with this interpretation, the data, in the present form, need to be adjusted. The following technique is used to convert the phase angles to the form in which they will be interpreted. The equation used to obtain the adjusted phase angle is

$$\theta_{adj} = \theta_m - \theta_{ha} \tag{20}$$

where

θ_{adj} phase angle from the harmonic analysis adjusted for interpretation purposes, deg

θ_m correction factor equal to 360^o if θ_{ha} is positive or equal to 0^o if θ_{ha} is negative

With these adjusted phase angles, a correction factor for the spacing of the standard IRIG record and playback heads is obtained. The mechanical tolerances of these heads, each of which holds seven channels, allow the possibility of a slight timing shift in the adjacent channels of the recorded data to the playback data. This shift would cause an error in the relative phase-angle measurements, particularly at the higher frequencies. To calculate the correction factor, it is assumed that there is no shift in phase angle between all the signals used in calibrating the transducers. The adjusted phase angles are then used to calculate the time shift between the various channels from the following equation:

$$K_T = \frac{(\theta_{adj} - \theta_{min})p}{360} \tag{21}$$

where

K_T correction time for IRIG head tape offset, sec

θ_{min} minimum value of θ_{adj}, deg

p period of the calibration signal, sec

(2) Data averaged and converted to engineering units – The data are digitized in the identical sequence as that used in the AC calibrations; however, instead of just 9 cycles selected from a run as in the AC calibrations, each 9 cycles over the entire frequency range is grouped to form a data point. The frequency is computed from equation (8), the sampling rate of 12 samples per cycle is maintained and averaged as done in equation (9), and conversion from counts to nondimensional amplitude is shown in equation (10). The 12 averaged amplitudes are then converted to engineering units by using the following equation:

$$Q_0 = \frac{V_0 K_S}{G} \tag{22}$$

where

Q_0 amplitude of the first sample, engineering units

G gain setting for each channel

The amplitudes Q_1 through Q_{11} are obtained in like manner.

(3) Amplitude and phase-angle calculations – The Fourier coefficients of first cosine term a_1 and first sine term b_1 in harmonic analysis are obtained as described in AC calibrations of the calculation procedures except that the amplitudes of the 12 averaged samples for each channel are $Q_0, Q_1, \ldots, Q_{11}$ in engineering units instead of the nondimensional amplitudes $V_0, V_1, \ldots, V_{11}$. The amplitude of the fundamental frequency in engineering units is obtained by using equations similar to those used in determining the nondimensional amplitude of the fundamental frequency. These are equations (14), (16), (17), and (18). The phase angle is calculated by using equations (15) and (20). This phase angle is then corrected by using the following equation:

$$\theta = \theta_{adj} - \theta_{irig} + \theta_{orient} \tag{23}$$

where

θ_{irig} phase-angle correction for IRIG head tape offset

θ_{orient} phase-angle orientation correction, 0° or 180°

The phase-angle correction for the IRIG head tape offset θ_{irig} is determined from

$$\theta_{irig} = K_T 360 f \tag{24}$$

(4) Digitally filtered frequency, amplitude, and relative phase angle – The amplitude A and relative phase angle θ are converted to rectangular coordinates. Then the frequency and the rectangular coordinates are filtered with a 41-point low-pass filter having a termination frequency of 0.5 Hz and a cut-off frequency of 0.25 Hz. Although there are small deviations in the measured data when a sweep frequency test is run, it is assumed that the increments are constant during the filtering because the differences are small. The filter shape is shown in figure 7. The first 20 points at the beginning and the last 20 points at the end of the measured data are lost when using this 41-point filter. After filtering, the rectangular coordinates are converted back to amplitude and relative phase angle.

(5) Filtered normalized amplitude, referenced phase angle, and calculated $\Delta s/2f\Delta f$ – The normalized amplitude A_n is obtained from

$$A_n = \frac{A}{A_r} \tag{25}$$

where

A amplitude at the fundamental frequency of a signal, engineering units

A_r amplitude at the fundamental frequency of the normalizing signal, engineering units

The referenced phase angle ϕ is given by

$$\phi = \theta - \theta_r \tag{26}$$

where

θ relative phase angle, deg

θ_r relative phase angle of the reference channel, deg

To allow the angle ϕ to be interpreted as the angle by which the test channel leads the reference channel, a correction of 360° is added to ϕ if ϕ is negative. The polar coordinates A_n and ϕ are converted to rectangular coordinates X, the normalized component along the real axis, and Y, the normalized component along the imaginary axis. The expression $\Delta s/2f\Delta f$ is computed from

$$\frac{\Delta s}{2f\Delta f} = \frac{\sqrt{\left(X_i - X_{i-1}\right)^2 + \left(Y_i - Y_{i-1}\right)^2}}{2f_i\left(f_i - f_{i-1}\right)} \tag{27}$$

where Δs is the distance between consecutive data points. For each frequency data point, the amplitude, the relative phase angle, the normalized amplitude, the referenced phase angle, and the value of $\Delta s/2f\Delta f$ are calculated. This output is written on a binary tape with 498 words per record. The data are blocked 8 frames per record. For a complete description of this tape, see figure 8.

(6) Frequency response plots – As discussed in the section entitled "Basic Flow of Data Analysis Processing," the results of the initial data reduction are presented to the experimenter. Several options as to the form of the presentation are available. (See fig. A1.) The standard option automatically selects scales to fit the plots on 21.6 cm by 27.9 cm $\left(8\frac{1}{2}'' \text{ by } 11''\right)$ paper (fig. A11). The digital data are retained awaiting the decision of the experimenter concerning the data that will continue to be analyzed.

(7) Polar plots (optional) – As discussed in the section entitled "Basic Flow of Data Analysis Processing," an experimenter may select relatively narrow frequency ranges of interest to be plotted for review. This option is selected by marking the appropriate square on the output requirements form (fig. A1) and submitting a modal search frequency range request form (fig. A12). Examples of these plots are shown in figure A13.

(8) Calculation of resonant frequency and damping – Before the resonant frequency and damping can be determined, the modal circle must be fitted to the data. A maximum of 200 data points may be used in these calculations. The following steps describe the procedure used:

(a) Fit circle to data – In the specified frequency range, a circle is fitted to the normalized amplitude and referenced phase-angle data. The equation of a circle is

$$x^2 + y^2 + Cx + Dy + E = 0 \tag{28}$$

The coefficients C, D, and E are determined by a least-squares method. The coordinates of the center of the circle x_c, y_c are

$$x_c = -C/2 \tag{29}$$

$$y_c = -D/2 \tag{30}$$

The radius of a circle is

$$r = 0.5\sqrt{C^2 + D^2 - 4E} \tag{31}$$

and the resonant diameter of a circle is

$$d = \pm 2r \tag{32}$$

The sign of the resonant diameter is positive if the y coordinate of the maximum value of $\Delta s/2f\Delta f$ is greater than the y coordinate of the displaced origin and is negative if the y coordinate of the maximum value of $\Delta s/2f\Delta f$ is less than the y coordinate of the displaced origin.

(b) Force data points to circle – If necessary, the x,y coordinates of each data point are forced to the circle along a radial line from the center of the circle through the data point.

(c) Calculate resonant frequency and displaced origin – The resonant frequency f_m is the frequency at the maximum value of $\Delta s/2f\Delta f$. The displaced origin is the point on the circumference of the circle diametrically opposite the resonant frequency. The coordinates of the resonant frequency adjusted to the circle x_m,y_m, the coordinates of the center of the circle x_c,y_c, and the geometry of the circle are used to calculate the coordinates of the displaced origin x_d,y_d as follows:

$$x_d = 2x_c - x_m \tag{33}$$

$$y_d = 2y_c - y_m \tag{34}$$

(d) Calculate damping – Damping is calculated by use of equation (6). The change in phase angle on the resonant peak $\Delta\phi$ was selected to be 20^o ($\pm 10^o$ when measured from the displaced origin about the resonant diameter). Figure 9 shows the various parameters used in this calculation. The coordinates, and therefore the frequency, at which this angle intersects the circle are calculated by using the following geometrical relationships:

$$\tan\frac{\Delta\phi}{2} = \frac{g_2 - g_1}{1 + g_1 g_2} \tag{35}$$

$$g_1 = \frac{y_d - y_c}{x_d - x_c} \tag{36}$$

$$g_2 = \frac{y_d - y_1}{x_d - x_1} \tag{37}$$

where g_1 and g_2 are the slopes of the chord lines describing the damping angle and x_1 and y_1 are the coordinates of f_1. The simultaneous solution of equation (35) and the equation for the fitted circle, as determined by equation (28), yields the values of x_1 and y_1. The x_2,y_2 coordinates may be determined in a similar manner.

If the legs of the angle do not intersect a data point, a linear interpolation of the distance between points on the circle is used to calculate f_1 and f_2.

By substituting 20° into equation (6), the damping is computed from

$$\mu = \frac{5.72958(f_2 - f_1)}{f_m} \tag{38}$$

If the data do not include x_2, y_2 points, the damping is computed from

$$\mu = \frac{11.45916(f_m - f_1)}{f_m} \tag{39}$$

or, if the data do not include x_1, y_1 points, the damping is computed by

$$\mu = \frac{11.45916(f_2 - f_m)}{f_m} \tag{40}$$

The damping is noted by an asterisk if only 10° is used to calculate the damping factor.

(9) Circle-fitted resonant response data plots (optional) – As discussed in the section entitled "Basic Flow of Data Analysis Processing," an experimenter may request plots showing the best circle fitted to the resonant response data to aid in his evaluation of the quality of the interpretation. This option is selected by marking the appropriate squares on the output requirements form (fig. A1) and the modal search frequency range request form (fig. A14).

(10) Mode plots and tabulated listing of resonant frequency and damping – As was done in the section entitled "Basic Flow of Data Analysis Processing," the final results of this program are now presented.

(a) Mode plots – The largest value from equation (32) is used to normalize all other values from equation (32). These values are then plotted as a function of the normalized station values given on the form shown in figure A5. Examples of these plots are shown in figures A16 and A19.

(b) Tabulated listing of resonant frequency and damping – The results of the resonant frequency calculation (i.e., the frequency at the maximum value of eq. (27)) as well as the inertance at resonance (i.e., the results of eq. (32)), and the damping (i.e., the results of eq. (38)) are tabulated for each data channel and each resonant frequency. Examples of these results are shown in figures A17 and A20.

VARIATIONS OF THE PRIMARY PROGRAM

Although the basic program is designed to obtain mode shapes, resonant frequencies, and modal damping, the procedures for reducing the sinusoidal data may be used for other purposes. Two possible applications are to obtain (1) curves of amplitude as a function of frequency or (2) curves of transfer function as a function of frequency. The method of obtaining data for both these applications would be identical to that outlined herein; however, the data could represent any oscillatory parameter and not be limited to force and acceleration. Because the signals may be arbitrary, the $\Delta s/2f\Delta f$ curve could be meaningless and should not be calculated. The amplitude-frequency curves may be obtained simply by processing the data by using steps (1) through (6) and bypassing the normalization program. If there is no reference channel given, then the phase-angle calculation also has no significance. To obtain the transfer functions, a reference channel would have to be given and, again, the data would be processed by using steps (1) through (6). For this case, the phase angle would be calculated and should be considered part of the transfer function.

CONCLUDING REMARKS

This report provides a guide to the use of the Sinusoidal-Vibration Analysis Program and to experimental procedures for obtaining data to be analyzed by that program. The program is a general purpose program for the analysis of structural vibration data which provides modal amplitudes, resonant frequencies, and modal damping values of a test structure. This report outlines the method of Kennedy and Pancu for obtaining resonant frequencies and modes, presents the phase-change method of calculating damping, establishes requirements for obtaining experimental data, presents the data analysis processing methods, and outlines some available variations of the basic program.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., June 4, 1973.

APPENDIX

SAMPLE CASE USING THE SINUSOIDAL-VIBRATION ANALYSIS PROGRAM

A sample case using the Sinusoidal-Vibration Analysis Program is presented to aid an experimenter to understand the procedure used to obtain the modes, frequencies, and damping of a structure and to interpret these results at various stages in the program. A comparison, with analytically calculated results, of the results of this data analysis program and the results of the peak-amplitude method is presented at the end of this appendix.

The purpose of the example experiment is to determine the modes, frequencies, and damping of a structure over the frequency range from 44.7 Hz to 53.6 Hz. During the experiment, the input was from an electromagnetic shaker attached to one end of the structure and using a constant level force. The frequency of the force was increased at a rate of 0.2 octave per minute. Data were recorded on magnetic tape from the force gage and several accelerometers along the structure.

The completed forms submitted by the experimenter (input) and the results (output) of the program are included in this appendix. In general, all the instructions needed to complete the forms are included on the forms. It should be noted that although the request form includes an AC calibration run (run 100) and three data runs (runs 101 to 103), only the calibration run and the last data run (run 103) are presented. Also, only the results of three of the seven channels are given. The following five forms are completed and submitted to the data reduction section to initiate the Sinusoidal-Vibration Analysis Program:

1. INPUT – Output requirements form (fig. A1)
2. INPUT – Data transcription work request form (fig. A2)
3. INPUT – Frequency range of test form (fig. A3)
4. INPUT – Gain settings and calibration levels form (fig. A4)
5. INPUT – Channel identification form (fig. A5)

In addition to these forms, it is important that the data be taken in a manner compatible with this program. The following requirements serve as a reminder of the test procedures which experience has shown cause delays if not done properly. The first three requirements refer to the calibration signal.

(1) All calibrations must be AC and recorded on the same tape as the data runs to which they refer.

(2) The AC calibrations must be from a common oscillator, including reference channel 13, as they are used for both instrument sensitivities and the IRIG head offset

phase correction. It is preferred, although not necessary, to have the calibration signal at the highest frequency of the test. The voltage of the calibration signal should be between 0.5 to 0.8 times the peak voltage capability of the recorder.

(3) The AC calibrations must be coded as type 5 on the gain settings and calibration levels form (fig. A4) and the calibration level supplied should be the zero to peak value of the physical quantity being measured.

The next four requirements are part of the technique developed to identify data recorded on tape by use of a time code generator. The coding of this system uses the modified NASA 36-bit time code signal which should be recorded on channel 14 of the tape recorder(s).

(4) The test number (figs. A3, A4, and A5) refers to a test configuration and should remain the same as long as the test conditions are the same. Although this number may be logged on a calibration run, it does not affect the use of this calibration signal for any test configuration.

(5) The run number (Figs. A3, A4, and A5) refers to each segment of tape on which something has been recorded. All runs on tape, including calibration signals, data signals, false starts, and errors, should be assigned an individual number.

(6) The group number (figs. A3, A4, and A5) refers to each grouping of 12 data channels. The outputs of as many transducers as are requested may be analyzed; however, they must be first combined in groups of 12 or less and then calibrated and recorded as a unit.

(7) The time code generator signal, channel 14, indicating start and ending of a run must be logged on the frequency range of test form (fig. A3).

The last six requirements refer to general procedures for taking of data.

(8) Any channel that is to be ignored must have a 999 – a coded number for the data processing group – on the gain settings and calibration levels form (fig. A4).

(9) The test number, run number, and group number should be recorded on the voice channel as a guide when playing back the tape.

(10) Data runs must be coded as type 4 on the gain settings and calibration levels form (fig. A4), and the variation of any amplifier from its calibration level position is recorded on this form as gain setting.

(11) A reference signal, between 0.5 and 0.8 times the peak voltage capability of the tape recorder, should be recorded on channel 13. This signal must have the identical frequency of the forcing function since it is used to calculate the frequency of the data.

(12) If the program is to plot the mode shape and calculate the resonant frequency and damping, then the input force signal must be recorded on tape and selected as the normalizing channel.

(13) The relative phase-angle output of the transducers should be electrically identical and only corrections for orientation should be necessary (0° or 180°).

After the tape has been digitized, the experimenter will receive confirmation in the following two forms:

6. OUTPUT – ADTRAN operations log (fig. A6)

Included on this form is the following information:

(a) digital tape number

(b) ADTRAN operator's name or initials (this is helpful if questions are necessary about the digitizing)

(c) serial number for –
discriminator runs
AC calibration runs
data runs

(d) tape information
number of channels per frame
number of frames per block
number of blocks

(e) remarks and/or identification

7. OUTPUT – Data transcription work sheet (fig. A7)

This form contains information which will be used if a question about digitizing the data arises.

At this time the data are in the proper format for the vibration analysis to begin. As the analysis proceeds, the following three forms are supplied to the experimenter to allow him to check the input values as they appear in the program.

8. OUTPUT – AC calibration test constants and computed data (fig. A8)

This information is supplied to the experimenter and should be checked. The first six lines contain the input:

Line 1 contains the initials of the operator processing the program, a test identification label, and the date.

Lines 2 to 4 contain information for the operator to run the program and may be ignored by the experimenter. This information will be used if a question about the data arises.

Lines 5 and 6 contain the input from the gain settings and calibration levels form supplied by the experimenter. This input includes the serial number, run number, and group number. The coded run type should be 5 for AC calibration runs and the calibration level supplied for each channel. A check should be made to see that these values have been put in correctly.

The remaining lines contain the output:

Line 7 contains the cut-off frequency, in hertz, of the low-pass ADTRAN filter used (FILTER), the ratio of recorded speed to playback speed (SPEED) (Note: To obtain the real-time cut-off frequency of the data, multiply FILTER times SPEED), the summation of the period (XPER), the number of samples calculated (PTSCYC, which should always be 108), and the AC frequency calculated (ACFREQ). The experimenter should check the AC frequency; the other parameters are necessary only if questions arise later.

Line 8 contains the serial number, run number, group number, and minimum and maximum phase angles of all channels calibrated for this group.

Lines 9 to 16 contain the amplitude (AMPLITUDE), the phase angle (PHASE), the sensitivity constant calculated (SENSITIVITY), the IRIG head time constant (T OFFSET), and the delta phase angle (P OFFSET) for each channel. A check on the time offset and amplitude columns may be made to ensure that the data have been properly recorded. Time offset generally is no greater than 300 μsec and may be considerably less. The listed amplitude values are the calibration voltages given in percent of the voltage required to drive the carrier frequencies to the band edge times a factor of 10.

9. OUTPUT – Quantity test constants and computed data (fig. A9)

This information will be supplied and should be checked by the experimenter. The first 10 lines contain the input:

Lines 1 to 8 contain the serial number, run number, group number, channel number, sensitivity constant, and constant for correction of IRIG head offset for each channel.

Lines 9 and 10 contain the input from the gain settings and calibration levels form supplied by the experimenter. This input includes the serial number, run number, and group number. The coded run type should be 4 for data runs and the gain setting supplied for each channel.

The remaining lines contain the output:

Lines 11 and 12 contain the test number, run number, group number, serial number, frequency cut-off (FILTER), ratio of recorded speed to playback speed (SPEED), and the last frequency calculated (FREQ).

Lines 13 to 25 contain the minimum and maximum values, in engineering units, for each channel. The magnitude of these values is used if questions arise. A *9999999. is tabulated if the channel is not used.

Line 26 contains the number of frequencies (FRAMES) calculated.

10. OUTPUT – Filtered normalized amplitude and phase-angle calculations test constants, and summary of data computed (fig. A10)

The first 13 lines contain the input:

Lines 1 to 13 contain the input from the channel identification form. If the channel identification is different for each serial number, the serial number will also be listed. Because the channel identification is the same for all serial numbers in this problem, no serial number has been listed.

The remaining lines contain the output:

Line 14 contains the number of points calculated, the test number, the serial number, and the frequency range calculated.

Lines 15 to 28 contain the minimum and maximum values of the amplitude (AMPLITUDE), phase angle (PHASE), normalized amplitude (NOR AMP), referenced phase angle (NOR PA), and the $\Delta s/2f\Delta f$ (DS/2FDF) calculation for each channel.

Lines 29 to 33 contain the number of times a channel was overscale. If a channel was not used, the number should be the total number of points.

The data are on a magnetic tape. The plot options selected are now implemented.

11. OUTPUT – Frequency response plots (fig. A11)

As requested on the output requirements form, the logarithm of the $\Delta s/2f\Delta f$ function, the referenced phase angle, and the logarithm of the normalized amplitude are plotted as a function of frequency. The short markers along the frequency scale identify blocks of 100 data points to aid the experimenter in selecting frequency ranges, since he must remember that for polar plots no more than 500 points may be plotted and for mode plots no more than 200 data points may be plotted.

The response of channel 2 shows two peaks (fig. A11(a)); however, the two peaks are not as visible on channels 5 (fig. A11(b)) and 7 (fig. A11(c)). The $\Delta s/2f\Delta f$ plots clearly show two modes on channels 2 and 7 and indicate a second mode on channel 5. The two modes occur between 48 and 51 Hz, but it is difficult to determine from these data how they contribute to each other's response. Therefore, a request for the data plotted in polar form is made to aid in selecting the frequencies to separate the modes. To perform this operation, the following form is filled in which lists the beginning and end frequencies of this limited frequency range of investigation:

12. INPUT – Modal search frequency range request form (fig. A12)

This form is checked for the polar plot option.

13. OUTPUT – Polar plot (fig. A13)

Now it can be clearly seen from the two circular-arc segments that there are two modes in this frequency range as was indicated by the $\Delta s/2f\Delta f$ plots.

An updated modal search frequency range request form separating the two frequency ranges, one frequency range for each mode, is now submitted, and the mode shape plots and damping and resonant frequency tables are obtained. As a guide in selecting the frequency range of the circular-arc segments which represent modes, five frequencies are presented on the polar plots near the data points they represent. Also requested in this example are the circle-fitted resonant response data plots and the detailed plot of $\Delta s/2f\Delta f$ as a function of frequency for each of the channels.

14. INPUT – Modal search frequency range request form (fig. A14)

This form is checked for mode plots and for circle-fitted resonant response data plots.

15. OUTPUT – Circle-fitted resonant response data plot and plot of $\Delta s/2f\Delta f$ against frequency for run 103, mode 1 (fig. A15)

16. OUTPUT – Mode plot for run 103, mode 1 (fig. A16)

17. OUTPUT – Tabulated resonant frequency and damping for run 103, mode 1 (fig. A17)

18. OUTPUT – Circle-fitted resonant response data plot and plot of $\Delta s/2f\Delta f$ against frequency for run 103, mode 2 (fig. A18)

19. OUTPUT – Mode plot for run 103, mode 2 (fig. A19)

20. OUTPUT – Tabulated resonant frequency and damping for run 103, mode 2 (fig. A20)

Superimposed on the modal amplitude plots from this program are (1) the modal amplitudes determined from the peak amplitude plots and (2) the calculated mode shapes (fig. A21). The calculated modes for this example are (1) a rigid body displacement mode at 49.0 Hz and (2) a rigid body rotation mode at 50.1 Hz. The program results show good agreement for both the amplitude values and general mode shape for the first mode with only a slight rotation about one end indicated. The amplitude values for the second mode bracket the actual mode amplitudes, and the general mode shape is in good agreement with the calculated values. If the peak amplitudes had been used to plot the mode shapes, the mode at 49.0 Hz appears to have elastic motion and the mode at 50.1 Hz has a nodal point at a location 10 percent of the beam length to the right of the actual nodal point for that mode. In addition, for the second mode, the amplitude values are higher than those calculated by about 0.4 for all the observable values and the values at stations 0.6 and 0.8 are not even observable as peaks in the data at the resonant frequency.

SHEET 1

DATE 9-1-72

SINUSOIDAL-VIBRATION ANALYSIS PROGRAM
OUTPUT REQUIREMENTS

JOB TITLE ANALOG COMP. MODEL TEST NUMBER 238 JOB ORDER RLK161

ENGINEER J SCHUENSTER PHONE 3451 ACCOUNT NO. 800305

FREQUENCY RANGE 45-54 TASK NO.* 32445
*(FILLED IN BY ACD)

I. PLOTTING DEVICE

☐ VARIAN PLOTS (WORKING PLOTS)

☒ CALCOMP (REPORT PLOTS)

☒ BLANK PAPER
☐ GRID PAPER ☐ 10 DIV/IN ☐ 20 DIV/IN ☐ RED RED ONLY ☐ BLUE ☐ GREEN

II. OUTPUT PLOTS

A. FREQUENCY RESPONSE PLOTS

☒ ONE CHANNEL PER PAGE (STANDARD) LOG $\Delta S/2F\Delta F$
REFERENCED PHASE ANGLE
LOG NORMALIZED AMPLITUDE
VS FREQUENCY

☐ NONSTANDARD FREQUENCY RESPONSE PLOT
(MUST SPECIFY ON SHEET 2)

B. MODE PLOTS AND TABULATED LISTING OF RESONANT FREQUENCY AND DAMPING

☒ POLAR PLOTS (NORMALIZED AMPLITUDE VS REFERENCED PHASE ANGLE)

☒ MODE PLOTS (NORMALIZED DIAMETER VS NORMALIZED STATION)
AND TABULATED LISTING OF RESONANT FREQUENCY AND DAMPING

☒ CIRCLE-FITTED RESONANT RESPONSE DATA PLOTS

☐ NO MODE PLOTS OR TABULATED LISTING OF RESONANT FREQUENCY AND DAMPING

Figure A1. - Output requirements form.

SHEET 2

SINUSOIDAL-VIBRATION ANALYSIS PROGRAM
OUTPUT REQUIREMENTS

DATE__________

JOB TITLE______________________ TEST NUMBER__________________________
ENGINEER______________________

II. OUTPUT PLOTS

A. FREQUENCY RESPONSE PLOTS

NONSTANDARD FREQUENCY RESPONSE PLOT

X AXIS

FREQUENCY	☐ LINEAR	FREQUENCY SCALE OR FREQUENCY AXIS	_____HZ PER INCH _____INCHES
	☐ LOG	FREQUENCY SCALE	_____OCTAVES PER INCH

Y AXIS

☐ AMPLITUDE	☐ LINEAR	☐ LOG	3 INCHES
☐ PHASE ANGLE	☐ LINEAR		2 INCHES
☐ NORMALIZED AMPLITUDE	☐ LINEAR	☐ LOG	3 INCHES
☐ REFERENCED PHASE ANGLE	☐ LINEAR		2 INCHES
☐ $\Delta S/2F\Delta F$	☐ LINEAR	☐ LOG	3 INCHES

Figure A1.- Concluded.

APPENDIX

DATA TRANSCRIPTION
WORK REQUEST

DATE 9-1-72

I. GENERAL

PROJECT OR TEST TITLE Analog Comp Model TEST NO. 238 TEST DATE 9-1-72

PROJECT ENGINEER J Schoenster BLDG. 1293B ROOM 208 M.S. 230 PHONE 3451

ORGANIZATION Loads Division J.O. NO. RLK 161 ACCOUNT NO. 8DD305

INSTRUMENTATION ENG. J Schoenster PHONE 3451 RECORDING STATION Langley

II. ANALOG TAPE

TAPE IDENTIFICATION 238 ORIGINAL ☒ COPY ☐

RECORDING HEADS: IRIG ☒ NON-IRIG ☐ RECORDED SPEED 7½ IPS

TYPE OF RECORDING: DIRECT ☐ SINGLE CARRIER FM ☒ FM/FM ☐

CONTINUOUS ☒

COMMUTATED ☐ PAM ☐ PDM ☐ PPM ☐ PCM ☐ CH/FR ______ FR/SEC ______

FULL SCALE SEGMENT NO. ______ ZERO SEGMENT NO. ______

TIME CODE TYPE NASA 36 BIT Modified TRACK NO. 14 DIRECT ☒ FM ☐

VOICE ANNOTATION ☒ TRACK NO. Edge Track DIRECT ☒ FM ☐

REFERENCE FREQ. (WOW AND FLUTTER COMPENSATION) ______ H_Z TRACK NO. ______

TAPE CALIBRATION ______ START ______ ______ STOP ______

		DAYS - HR - MIN - SEC	DAYS - HR - MIN - SEC
AC	☒	See Frequency Range of Test	Form
DC	☐		
AMBIENT	☐		

ZERO (LAUNCH TIME) ______
DAYS HOURS MIN. SEC. MILLI-SEC

III. TYPE OF TRANSCRIPTION

DIGITAL ☒ CONTINUOUS DATA SAMPLE RATE 12 Samples/Cycle SPS

COMMUTATED EVERY FRAME ☐ OTHER ______

TYPE OF ANALYSIS: E.U. ONLY ☐ TSA ☐ SIN-SWEEP ☒ OTHER ______

OSCILLOGRAPH ☐ PAPER SPEED ______ IPS TIMING LINES ☐

TAPE DUPLICATION ☐ DIRECT COPY ☐ OTHER ______

SPECTRAL CHARTS ☐ PSD ☐ ASD ☐ FREQ. RANGE ______ H_Z

(a) Tape information.

Figure A2. - Data transcription work request form.

____________CDT USE ONLY____________

CDT TAPE NO.____________ TASK NO.____________CALIBRATION

CONSTANTS____________ SPECIAL INSTRUCTION____________

COMPLETED OPR____________ DATE____________

(b) Data transcription log.

Figure A2.- Continued.

TAPE TRACK & CHANNEL ASSIGNMENT

TAPE TK	CH. NO.	CENTER FREQ & DEVIATION	FILTER SIZE	INSTRUMENT
1	1	13.5 kHz ±40%		FORCE
2	2			ACC 0
3	3			ACC 0.2
4	4			ACC 0.4
5	5			ACC 0.6
6	6			ACC 0.8
7	7			ACC 1.0
13	13			REF. SIGNAL
14	14	DIRECT		TIME CODE

DATA RUN INTERVALS

RUN NO.	START TIME			STOP TIME		
	HOUR	MIN.	SEC.	HOUR	MIN.	SEC.
(SEE	FRE	QUENC	Y RAN	GE OF	TEST	FORM)

NOTE

1. Use Additional Sheets As Required Where A Higher Number Of Channels Are Used Or An Increased Number Of Runs Are To Be Transcribed.
2. Attach Run Logs Or Other Specific Conversion Information.

SPECIAL INSTRUCTIONS ____________________

(c) Tape track channel assignment and data run intervals.

Figure A2. - Concluded.

APPENDIX

SINUSOIDAL-VIBRATION ANALYSIS PROGRAM
FREQUENCY RANGE OF TEST

DATE 9-1-72

JOB TITLE Analog Comp Model TEST NUMBER 238 JOB ORDER RLK 161

ENGINEER J Schoenster PHONE 3451 ACCOUNT NO. 800305

TEST	RUN	GROUP	FREQUENCY START	FREQUENCY STOP	TIME CODE ABCDEFXXX START	TIME CODE ABCDEFXXX STOP	REMARKS
238	100	01	50		0	50	AC CALIBRATION
238	101	01	45	53	18	118	NASA 36 BIT TIME CODE (MODIFIED)
238	102	01	45	54	30	130	ABC - TEST NUMBER
238	103	01	45	54	30	130	DEF - RUN NUMBER

Figure A3.- Frequency range of test form.

SINUSOIDAL-VIBRATION ANALYSIS PROGRAM
GAIN SETTINGS AND CALIBRATION LEVELS

DATE 9-1-72

JOB TITLE Analog Comp Model TEST NUMBER 238 JOB ORDER RLK 161

ENGINEER J. Schoenster PHONE 3451 ACCOUNT NO. 800305

	SN*	RUN	GR	TYPE	GAIN SETTING OR CALIBRATION LEVEL											
					CH1	CH2	CH3	CH4	CH5	CH6	CH7	CH8	CH9	CH10	CH11	CH12
1	5	8	10	11	12 16	17 21	22 26	27 31	32 36	37 41	40 46	47 51	52 56	57 61	62 66	67 71
D	5615	100	01	5	10.0	10.0	10.0	10.0	10.0	10.0	10.0	999.	999.	999.	999.	999.
D																
D	5616	101	01	4	1.0	1.0	1.0	1.0	1.0	1.0	1.0	999.	999.	999.	999.	999.
D	5617	102	01	4	1.0	1.0	1.0	1.0	1.0	1.0	1.0	999.	999.	999.	999.	999.
D	5618	103	01	4	1.0	1.0	1.0	1.0	1.0	1.0	1.0	999.	999.	999.	999.	999.
D																
D																
D																
D																
D																
D																
D																
D																
D																

SN* SERIAL NUMBER (THIS IS FILLED IN BY ACD)
RUN RUN NUMBER
GR GROUP NUMBER (IF ONLY 1 GROUP USE 01)
TYPE TYPE 4 = DATA RUN TYPE 5 = AC CALIBRATION RUN
CH GAIN SETTING FOR TYPE 4 DATA RUNS
CALIBRATION LEVEL FOR TYPE 5 AC CALIBRATION RUNS
IF A CHANNEL IS NOT USED - USE 999.0

Figure A4.- Gain settings and calibration levels form.

SINUSOIDAL-VIBRATION ANALYSIS PROGRAM
CHANNEL IDENTIFICATION

DATE 9-1-72

JOB TITLE Analog Comp Model TEST NUMBER 238 JOB ORDER RLK 161

ENGINEER J. SCHOENSTER PHONE 3451 ACCOUNT NO. 800305

NOTE: THESE CONSTANTS SHOULD BE USED FOR RUNS 101, 102, 103

	SN*	RUN	GR	CH	NOR CH	NOR STA	PLT SYM	PHASE CORR.		CHANNEL DESCRIPTION	
1	5	8	10	12	14	20	22	23	27	31	80
2	5616	101	01	01	1	.00	0		0.	INPUT FORCE STATION	
2	5616	101	01	02	1	.00	1		0.	NORMALIZED STATION 0	
2	5616	101	01	03	1	.20	1		0.	NORMALIZED STATION .2	
2	5616	101	01	04	1	.40	1		0.	NORMALIZED STATION .4	
2	5616	101	01	05	1	.60	1		0.	NORMALIZED STATION .6	
2	5616	101	01	06	1	.80	1		0.	NORMALIZED STATION .8	
2	5616	101	01	07	1	1.00	1		0.	NORMALIZED STATION 1.0	
2	5616	101	01	08	0	.00	0		0.		
2	5616	101	01	09	0	.00	0		0.		
2	5616	101	01	10	0	.00	0		0.		
2	5616	101	01	11	0	.00	0		0.		
2	5616	101	01	12	0	.00	0		0.		

SN* SERIAL NUMBER (THIS IS FILLED IN BY ACD)
RUN RUN NUMBER
GR GROUP NUMBER
CH CHANNEL NUMBER (ALL 12 CHANNELS MUST BE INCLUDED)
NOR CH NORMALIZING CHANNEL NUMBER (IF A CHANNEL IS NOT USED, THE NOR CH SHOULD BE 0)
NOR STA NORMALIZED STATION
PLT SYM SYMBOL PLOTTING CODE - PLT SYM =
1 CIRCLE
2 SQUARE
3 DIAMOND
4 TRIANGLE
5 RIGHT TRIANGLE
6 QUADRANT
7 DOG HOUSE
8 FAN
9 LONG DIAMOND
10 HOUSE
11-20 SAME AS 1-10 EXCEPT PLUS ENCLOSED IN SYMBOL

PHASE CORR PHASE ANGLE TRANSDUCER ORIENTATION CORRECTION TO ADD TO MEASURED PHASE ANGLE 0° OR 180°

CHANNEL DESCRIPTION-TITLE USED ON MODE PLOTS

Figure A5.- Channel identification form.

APPENDIX

ADTRAN OPERATIONS LOG
DIGITAL TAPE REEL LOG

PROJ Analog Comp. Model DIGITAL TAPE REEL NO. 4-07-024

DATE: 9-8-72 JOB ORDER RLK 161 TASK NO. 32445

USERS NAME J Schoenster OPR CRL/JmF

SERIAL NUMBER	CH/ FR	FR/ BLK	NO. OF BLOCKS	ANALOG REEL NO.	REMARKS/IDENTIFICATION	SAMPLE RATE
5614	16	01	15	—	DISC CALS SD = 0200000 FIL = 0000059	
5615	16	36	3	CDT 1790	AC CALS 50 Hz 2381001	
5616	16	36	1452	CDT 1790	DATA SWEEP 25.6-27.8 2381011	
5617	16	36	1650		DATA SWEEP 42.3-45.0 2381021	
5618	16	36	1656		DATA SWEEP 50.2-52.3 2381031	

NOTES

Figure A6.- ADTRAN operations log.

APPENDIX

DATA TRANSCRIPTION WORK SHEET

JOB TITLE *Analog Comp Model*

PROJ. ENG. *J Schoenster* PHONE *3451* BLDG. *1293B* RM. *208* M.S. *230*

ORG. CODE *22.840* J.O. NO. *RLK 161* TASK NO. *32445*

DATA TYPE: CONTINUOUS *X* COMMUTATED __ - CH. RATE *N/A* F.S. CH. *N/A* ZERO CH. *N/A*

CH/FR *16* FR/BLK *36* W+F COMPEN. FREQ. *N/A* TAPE TRACK *N/A*

WORD	UNITS	REMARKS
4		*DISC CAL LOAD LEVELS/AC CAL FREQ (TRUE SPEED)*
5		*TEST, RUN, AND GROUP*
6		*100 kHz PERIOD COUNTER*

WORD	TAPE TRACK	DISC. NO.	CENTER FREQ. AND DEVIATION	CUTOFF FREQ.	CA	CD	RECORDER CHANNEL	INSTRUMENT
11	*13*	*16*	*DIGITAL*	–	–	–	–	*NUMBER OF CYCLES*
	13	*16*	*WORD*	–	–	–	–	*NUMBER OF SAMPLES*
12	*1*	*2*	*6.75 kHz ±40%*	*59 Hz*				*FORCE*
	2	*3*						*ACC 0*
13	*3*	*4*						*ACC .2*
	4	*5*						*ACC .4*
14	*5*	*6*						*ACC .6*
	6	*7*						*ACC .8*
15	*7*	*8*						*ACC 1.0*
	8	*9*						*NOT USED*
16	*9*	*10*						*NOT USED*
	10	*12*						*NOT USED*
17	*11*	*13*						*NOT USED*
	12	*14*						*NOT USED*
18	*13*	*16*						*REF SWEEP FREQ*
	13	*16*						*REF SWEEP FREQ*
19								
20								

Figure A7. - Data transcription work sheet.

```
LINE 1          MPL C+S AP        ANALOG MOD        09/12/72

   2  CHECK PT PRINT   FILTER   SPEED   NUMBER  TAPE NUMBERS
   3   CODE    CODE                      TAPES
   4    0       0        0        0        0


   5  SN   RUN GR TYPE   CH 1      CH 2      CH 3      CH 4      CH 5      CH 6      CH 7      CH 8      CH 9      CH10      CH11      CH12
   6  5615 100  1   5    10.0000   10.0000   10.0000   10.0000   10.0000   10.0000   10.0000  999.0000  999.0000  999.0000  999.0000  999.0000


   7  FILTER   59     SPEED   2.00000     XPER    863244     PTSCYC      108     ACFREQ  5.00437883E+01

   8  SN 5615     RUN   100      GROUP     1      PHASE ANGLE MIN     303.95     PHASE ANGLE MAX     305.74

   9          CH             AMPLITUDE              PHASE         SENSITIVITY           T OFFSET                P OFFSET
  10           1             683.26691              303.95        1.46356E-02           0.                          0.00
  11           2             701.14037              304.66        1.42625E-02           3.92868E-05                  .71
  12           3             702.41156              304.69        1.42367E-02           4.08411E-05                  .74
  13           4             700.33752              305.06        1.42788E-02           6.12746E-05                 1.10
  14           5             693.93157              304.67        1.44106E-02           4.01692E-05                  .72
  15           6             684.67154              305.63        1.46055E-02           9.32898E-05                 1.68
  16           7             700.81097              305.74        1.42692E-02           9.91031E-05                 1.79
```

Figure A8. - AC calibration test constants and computed data.

```
LINE  1   SN  RUN  GR  CH  SENSITIVITY      IRIG HEAD OFFSET
      2  5615  100   1   1  1.46356E-02          0.
      3  5615  100   1   2  1.42625E-02          3.92868E-05
      4  5615  100   1   3  1.42367E-02          4.08411E-05
      5  5615  100   1   4  1.42788E-02          6.12746E-05
      6  5615  100   1   5  1.44106E-02          4.01692E-05
      7  5615  100   1   6  1.46055E-02          9.32898E-05
      8  5615  100   1   7  1.42692E-02          9.91031E-05

      9   SN  RUN GR TYPE   CH 1     CH 2     CH 3     CH 4     CH 5     CH 6     CH 7     CH 8     CH 9     CH10     CH11     CH12
     10  5618 103  1   4    1.0000   1.0000   1.0000   1.0000   1.0000   1.0000   1.0000 999.0000 999.0000 999.0000 999.0000 999.0000

     11    TEST      RUN    GRP    SN     FILTER       SPEED        FREQ
     12  2381031     103      1  5618         59     2.00000        53.6721

     13    CHAN        MIN              MAX
     14      1       -4.602833        4.850511
     15      2       -8.823000        8.655631
     16      3       -6.964387        7.349734
     17      4       -6.364120        6.626867
     18      5       -6.329948        6.178642
     19      6       -6.117874        6.262206
     20      7       -6.880360        6.924499
     21      8*9999999.000000*9999999.000000
     22      9*9999999.000000*9999999.000000
     23     10*9999999.000000*9999999.000000
     24     11*9999999.000000*9999999.000000
     25     12*9999999.000000*9999999.000000

     26  NO FRAMES        550
```

Figure A9.- Quantity test constants and computed data.

GR	CH	NOR CH	NOR STA	SYMBOL	PA CORR	DESCRIPTION
1	1	1	0.000	0	0	INPUT FORCE STATION
1	2	1	0.000	1	0	NORMALIZED STATION 0
1	3	1	.200	1	0	NORMALIZED STATION 0.2
1	4	1	.400	1	0	NORMALIZED STATION 0.4
1	5	1	.600	1	0	NORMALIZED STATION 0.6
1	6	1	.800	1	0	NORMALIZED STATION 0.8
1	7	1	1.000	1	0	NORMALIZED STATION 1.0
1	8	0	0.000	0	0	
1	9	0	0.000	0	0	
1	10	0	0.000	0	0	
1	11	0	0.000	0	0	
1	12	0	0.000	0	0	

NUMBER OF POINTS 504 TEST 2381031 SN 5618 FREQUENCY RANGE 44.75 TO 53.60

CH	AMPLITUDE MIN	AMPLITUDE MAX	PHASE MIN	PHASE MAX	NOR AMP MIN	NOR AMP MAX	NOR PA MIN	NOR PA MAX	DS/2FDF MIN	DS/2FDF MAX
1	4.976E+00	4.989E+00	3.107E+02	3.138E+02	1.000E+00	1.000E+00	0.	0.	0.	0.
2	1.155E+00	9.333E+00	9.052E-01	3.593E+02	2.317E-01	1.875E+00	8.625E+00	1.760E+02	-1.628E-03	3.413E-02
3	9.453E-01	7.558E+00	8.526E-01	3.593E+02	1.895E-01	1.517E+00	7.683E+00	1.759E+02	-1.816E-03	2.751E-02
4	7.335E-01	6.965E+00	2.222E-01	3.595E+02	1.471E-01	1.398E+00	9.040E+00	1.765E+02	-1.596E-03	2.874E-02
5	5.281E-01	6.579E+00	1.054E+00	3.592E+02	1.059E-01	1.320E+00	5.611E-02	3.595E+02	-1.578E-03	3.058E-02
6	8.202E-02	6.562E+00	1.421E+00	3.591E+02	1.647E-02	1.317E+00	1.815E-01	3.599E+02	-1.643E-03	3.258E-02
7	1.032E-01	7.367E+00	1.445E-01	3.577E+02	2.069E-02	1.480E+00	1.088E+00	3.591E+02	-2.008E-03	3.827E-02
8	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02
9	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02
10	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02
11	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02
12	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02	9.990E+02

CH 8 OVERSCALE 504

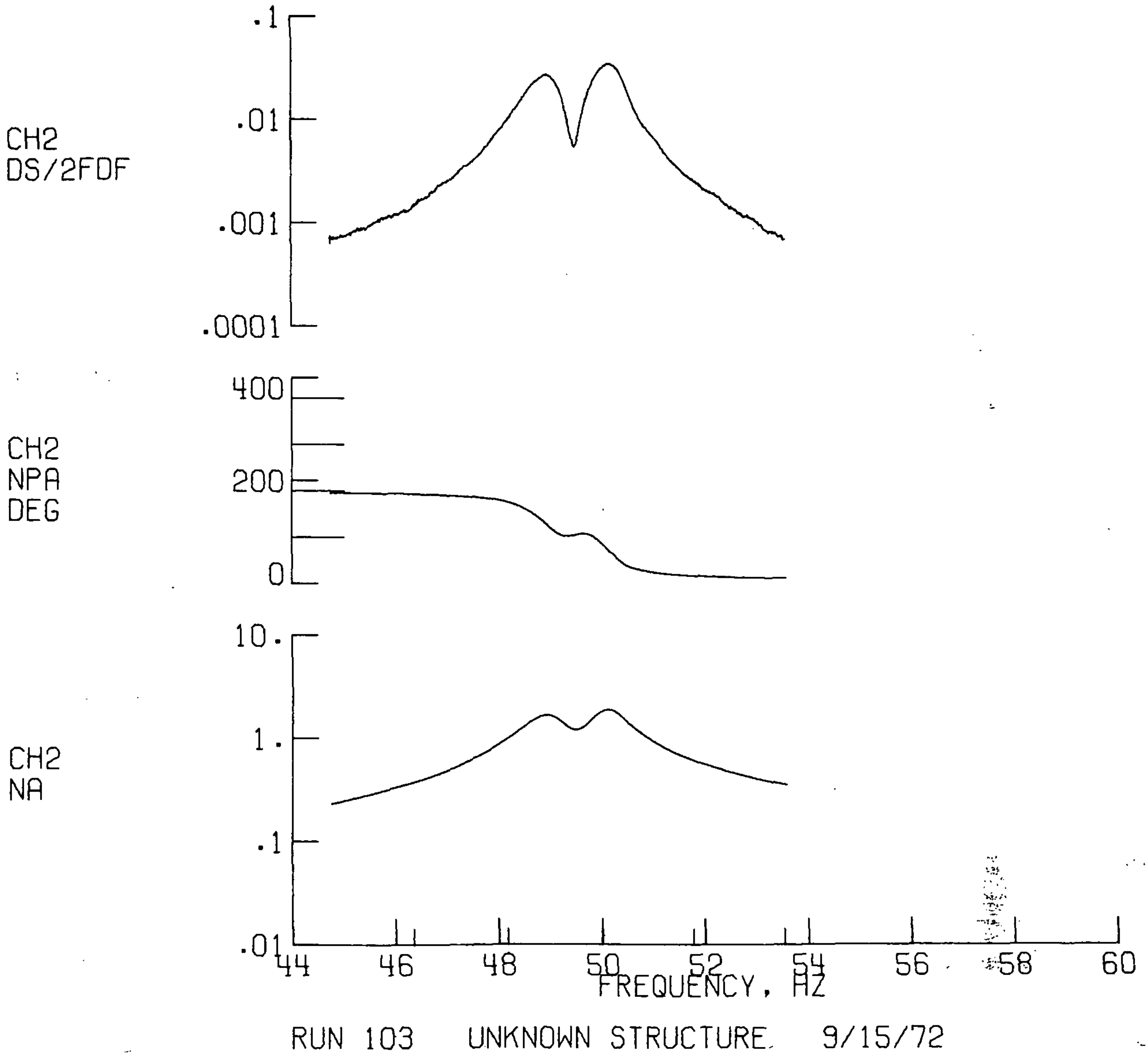
CH 9 OVERSCALE 504

CH 10 OVERSCALE 504

CH 11 OVERSCALE 504

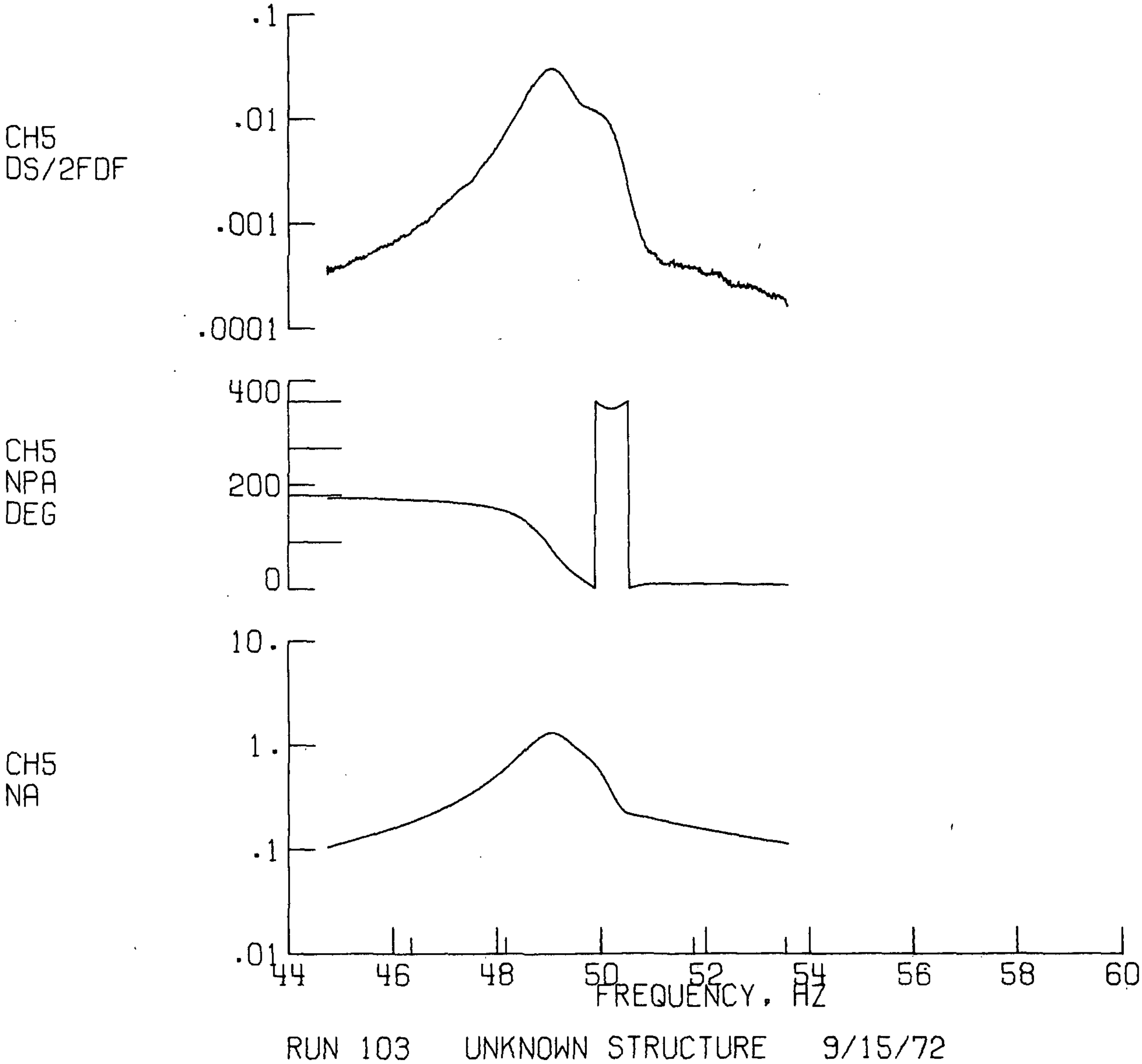
CH 12 OVERSCALE 504

Figure A10.- Filtered normalized amplitude and phase-angle calculations test constants and summary of data computed.



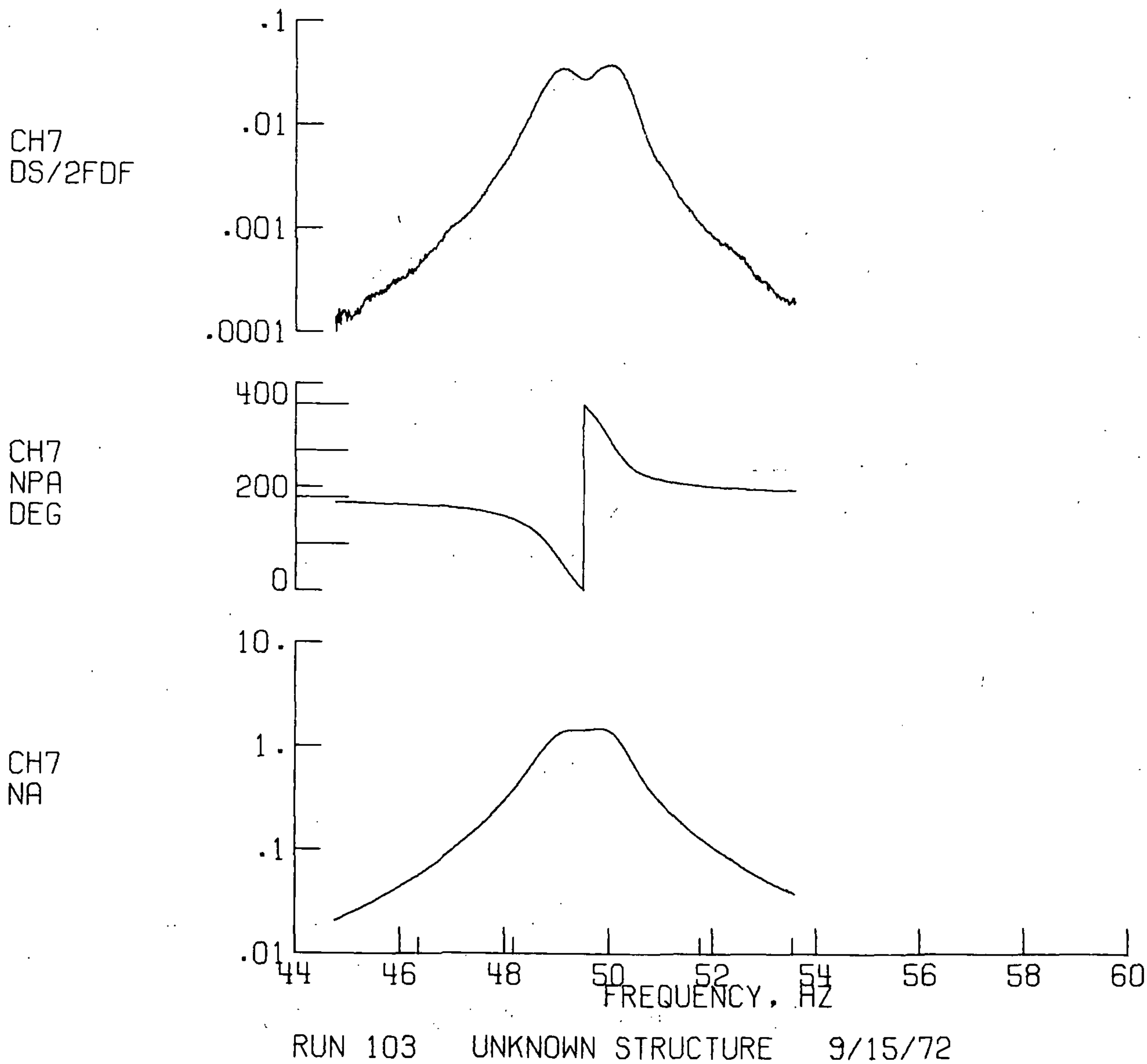
(a) Normalized station 0.

Figure A11.- Frequency response plots.



(b) Normalized station 0.6.

Figure A11.- Continued.



(c) Normalized station 1.0.

Figure A11. - Concluded.

SINUSOIDAL-VIBRATION ANALYSIS PROGRAM
MODAL SEARCH FREQUENCY RANGE REQUEST DATE 10/1/72

JOB TITLE Analog Comp Model TEST NUMBER 238 JOB ORDER RLK 161

ENGINEER J SCHUENSTER PHONE 3451 ACCOUNT NO. 800305

☒ POLAR PLOTS

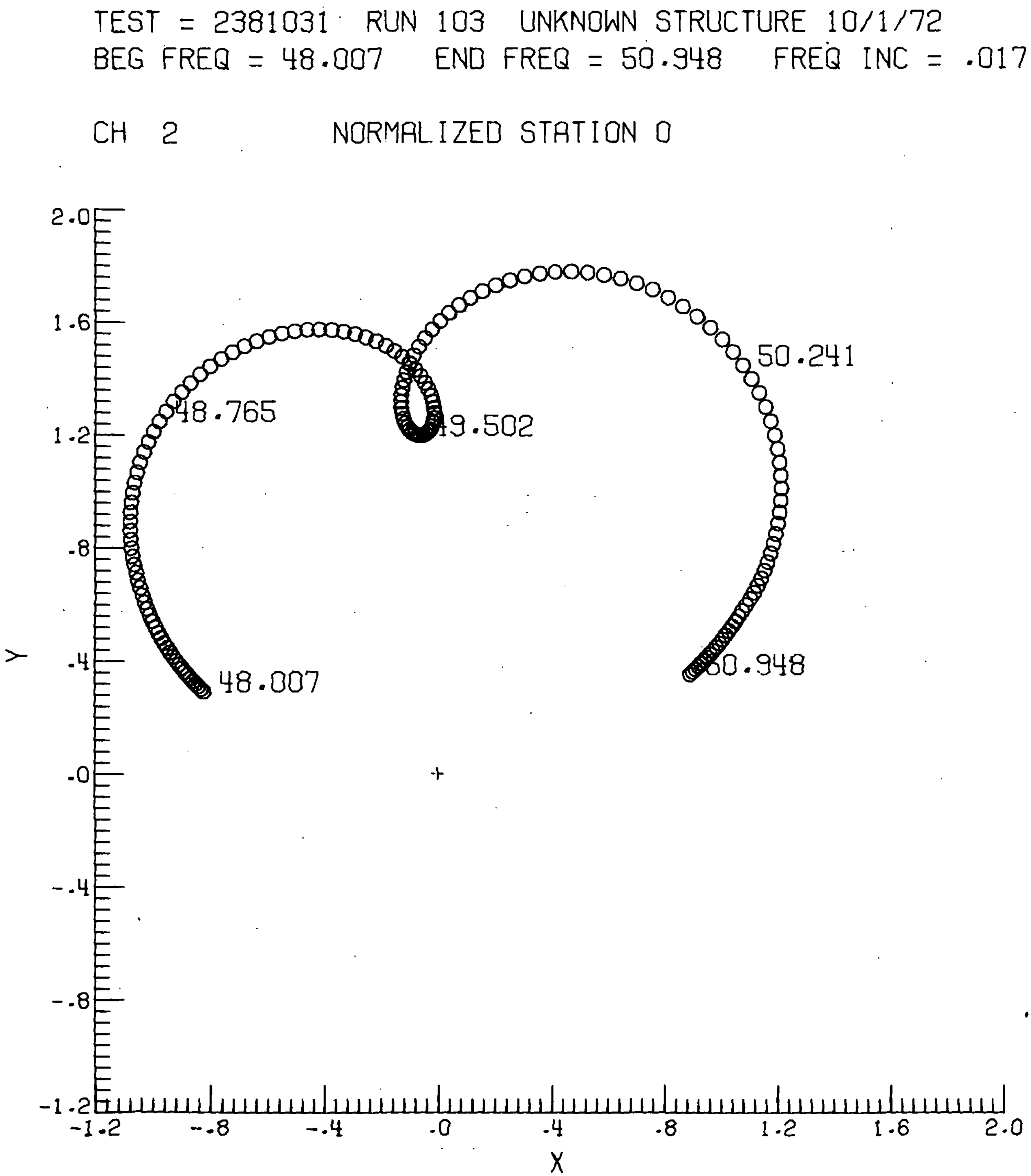
☐ MODE PLOTS

☐ CIRCLE-FITTED RESONANT RESPONSE DATA PLOTS

	SN	RUN	GR	BEG FREQ	END FREQ	RUN IDENTIFICATION
1	5	8	10	11 20	21 30	31 80
I	5618	103	01	48.00	50.96	RUN 103 UNKNOWN STRUCTURE 10/1/72
I						
I						
I						
I						
I						
I						
I						
I						
I						
I						
I						
I						
I						
I						
I						
I						
I						
I						

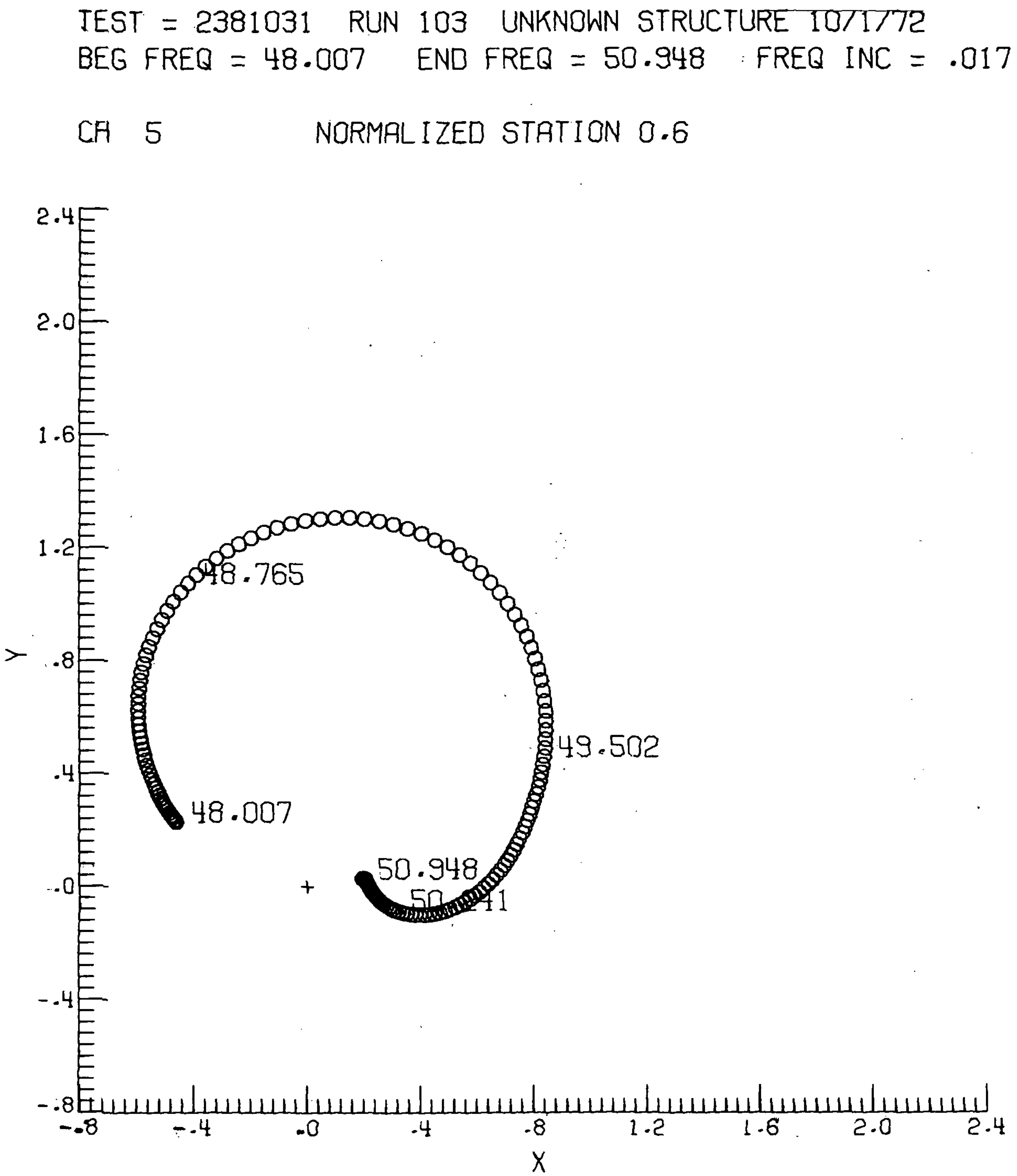
SN	SERIAL NUMBER
RUN	RUN NUMBER
GR	GROUP NUMBER
BEG FREQ	BEGINNING FREQUENCY
END FREQ	ENDING FREQUENCY
RUN IDENTIFICATION	TITLE USED ON PLOTS

Figure A12.- Modal search frequency range request form for polar plots.



(a) Normalized station 0.

Figure A13.- Polar plots.

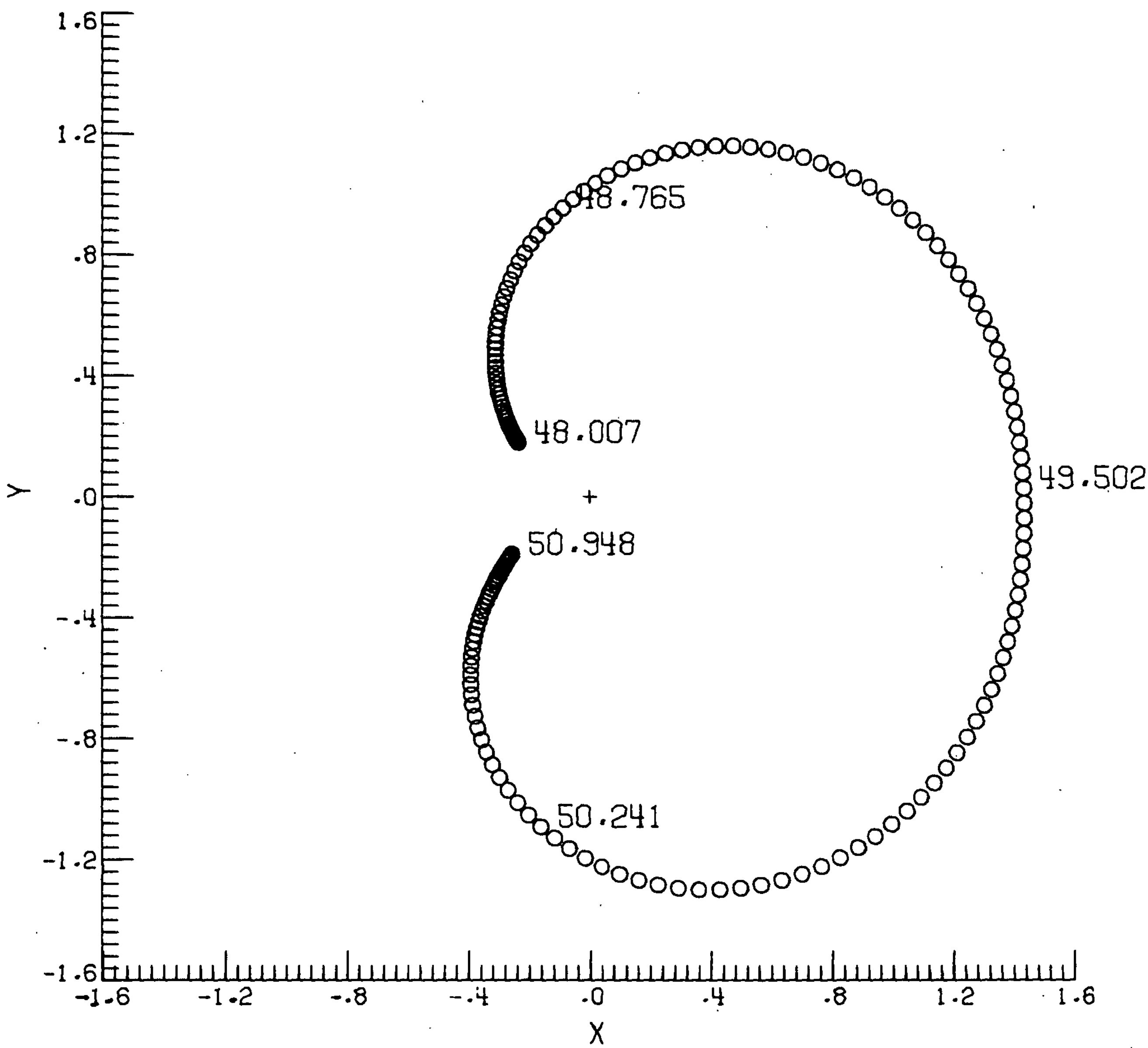


(b) Normalized station 0.6.

Figure A13.- Continued.

TEST = 2381031 RUN 103 UNKNOWN STRUCTURE 10/1/72
BEG FREQ = 48.007 END FREQ = 50.948 FREQ INC = .017

CH 7 NORMALIZED STATION 1.0



(c) Normalized station 1.0.

Figure A13.- Concluded.

APPENDIX

SINUSOIDAL-VIBRATION ANALYSIS PROGRAM
MODAL SEARCH FREQUENCY RANGE REQUEST DATE 10/1/72

JOB TITLE Analog Comp Model TEST NUMBER 238 JOB ORDER RLK 161

ENGINEER J SCHOENSTER PHONE 3451 ACCOUNT NO. 800305

☐ POLAR PLOTS

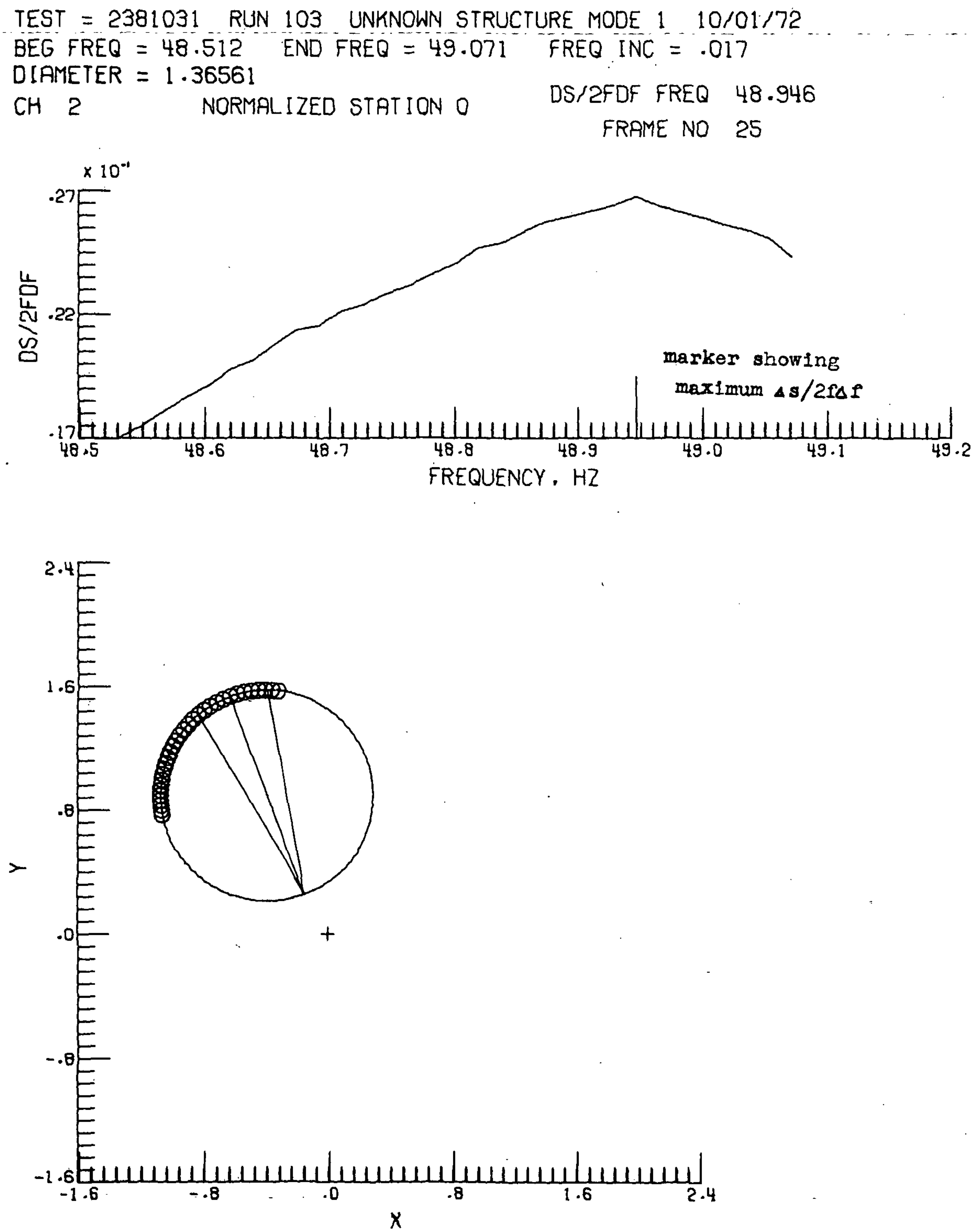
☒ MODE PLOTS

☒ CIRCLE-FITTED RESONANT RESPONSE DATA PLOTS

	SN	RUN	GR	BEG FREQ	END FREQ	RUN IDENTIFICATION
1	5	8	10	11 20	21 30	31 80
I	5618	103	01	48.51	49.08	RUN103 UNKNOWN STRUCTURE MODE 1 10/1/72
I	5618	103	01	49.91	50.48	RUN103 UNKNOWN STRUCTURE MODE 2 10/1/72
I						
I						
I						
I						
I						
I						
I						
I						
I						
I						
I						
I						
I						
I						
I						
I						
I						

SN	SERIAL NUMBER
RUN	RUN NUMBER
GR	GROUP NUMBER
BEG FREQ	BEGINNING FREQUENCY
END FREQ	ENDING FREQUENCY
RUN IDENTIFICATION	TITLE USED ON PLOTS

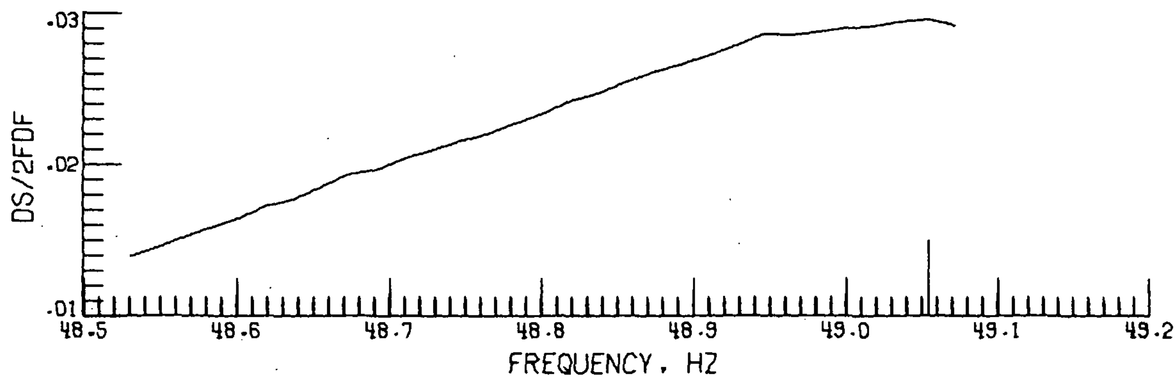
Figure A14.- Modal search frequency range request form for mode plots.

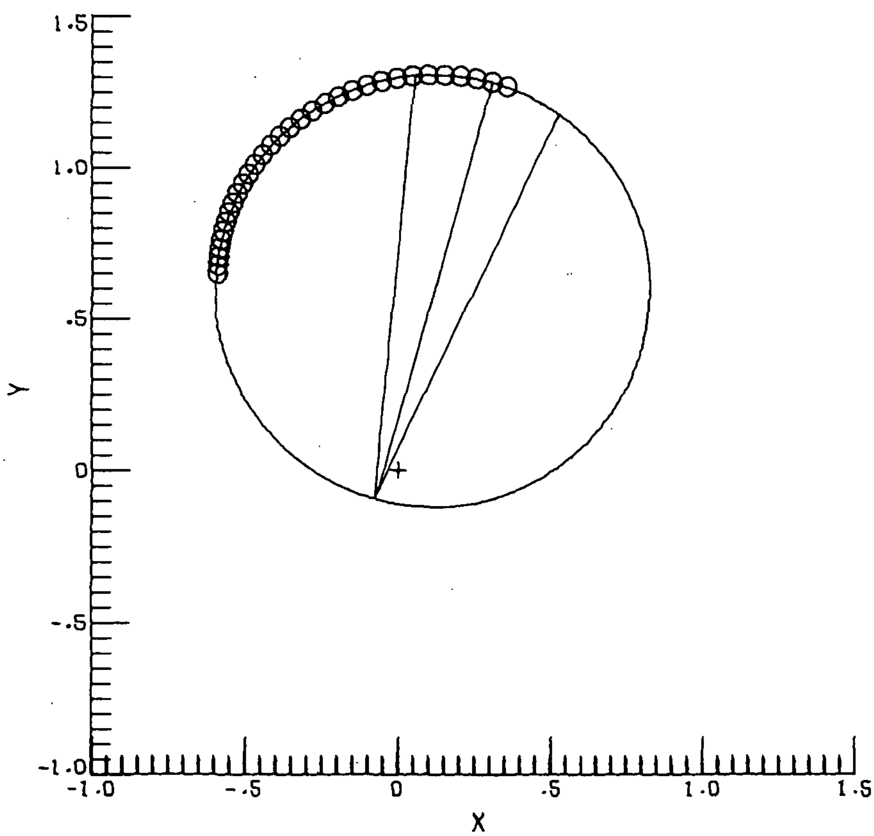


(a) Normalized station 0.

Figure A15.- Circle-fitted resonant response data plot and plot of $\Delta s/2f\Delta f$ against frequency for run 103, mode 1.

TEST = 2381031 RUN 103 UNKNOWN STRUCTURE MODE 1 10/01/72
BEG FREQ = 48.512 END FREQ = 49.071 FREQ INC = .017
DIAMETER = 1.42837
CH 5 NORMALIZED STATION 0.6 DS/2FDF FREQ 49.053
FRAME NO 31

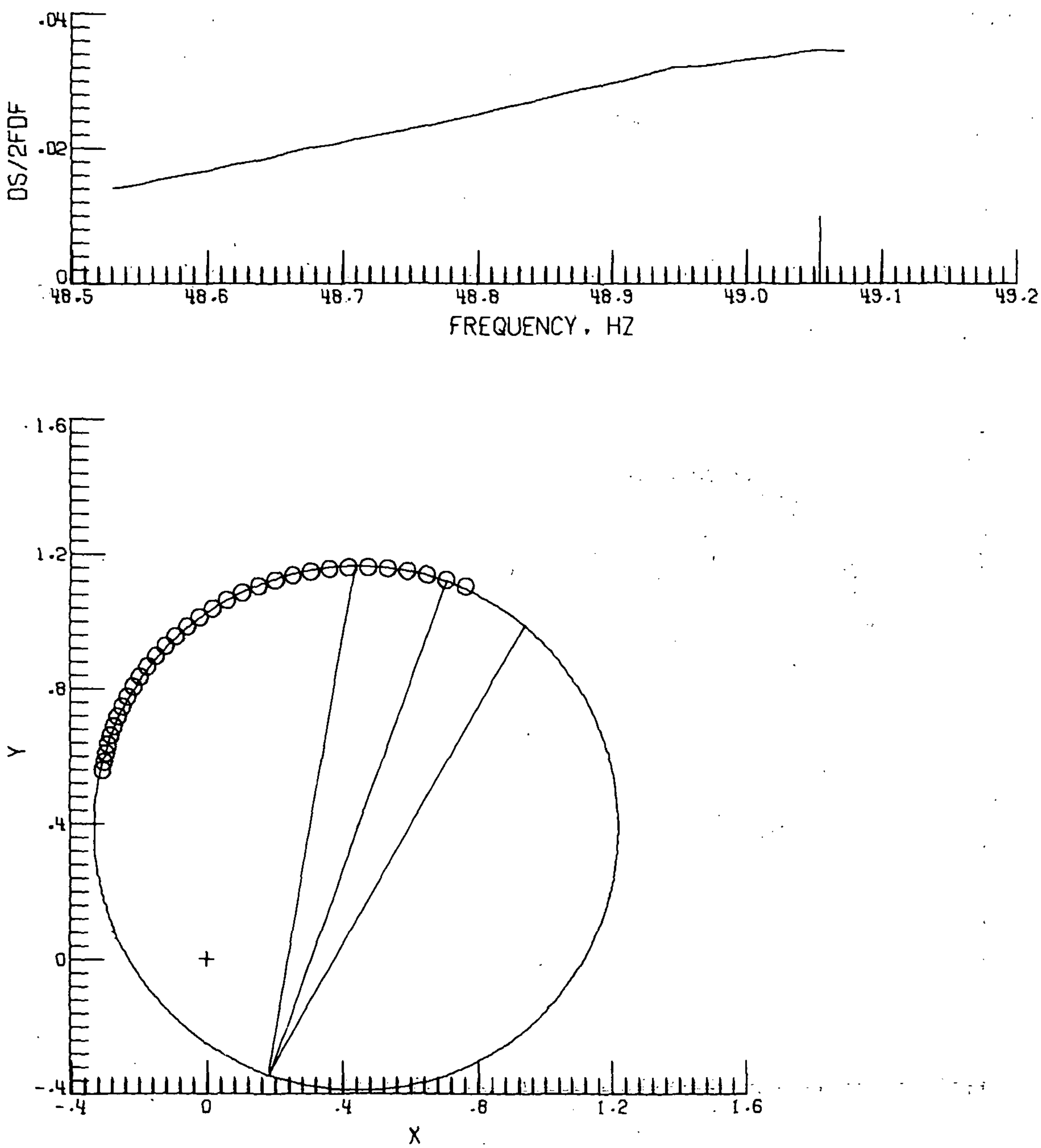




(b) Normalized station 0.6.

Figure A15.- Continued.

TEST = 2381031 RUN 103 UNKNOWN STRUCTURE MODE 1 10/01/72
BEG FREQ = 48.512 END FREQ = 49.071 FREQ INC = .017
DIAMETER = 1.55418
CH 7 NORMALIZED STATION 1.0 DS/2FDF FREQ 49.053
FRAME NO 31



(c) Normalized station 1.0.

Figure A15.- Concluded.

TEST = 2381031 RUN 103 UNKNOWN STRUCTURE MODE 1 10/01/72
BEG FREQ = 48.512 END FREQ = 49.071 FREQ INC = .017

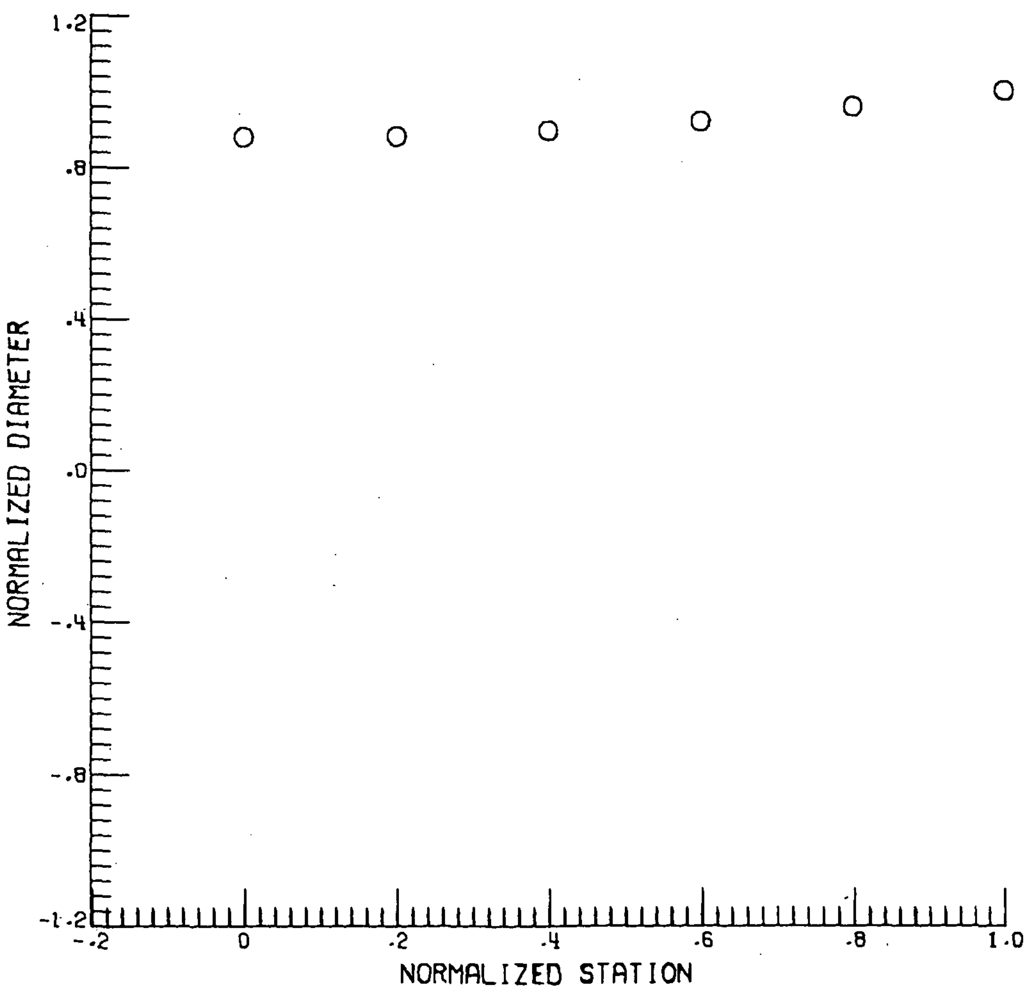


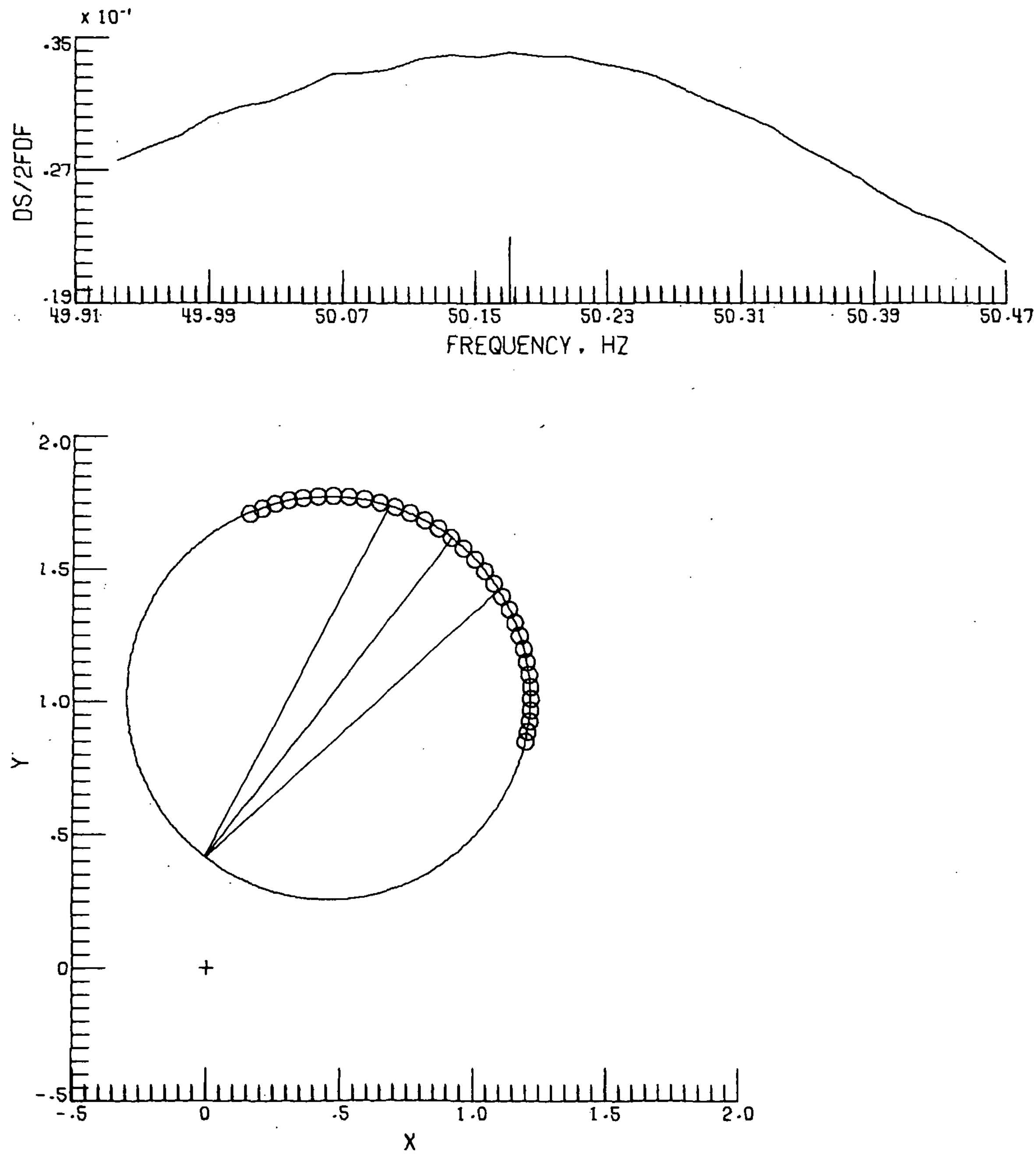
Figure A16.- Mode plot for run 103, mode 1.

TEST = 2381031 RUN 103 UNKNOWN STRUCTURE MODE 1 10/01/72
BEG FREQ = 48.512 END FREQ = 49.071 FREQ INC = .017

CH	NOR STA	FREQUENCY	DIAMETER	DAMPING
2	0.000	48.946	1.36561	.02194
3	.200	48.946	1.36918	.02126
4	.400	49.036	1.39229	.02029*
5	.600	49.053	1.42837	.01967*
6	.800	49.053	1.48902	.01937*
7	1.000	49.053	1.55418	.01912*

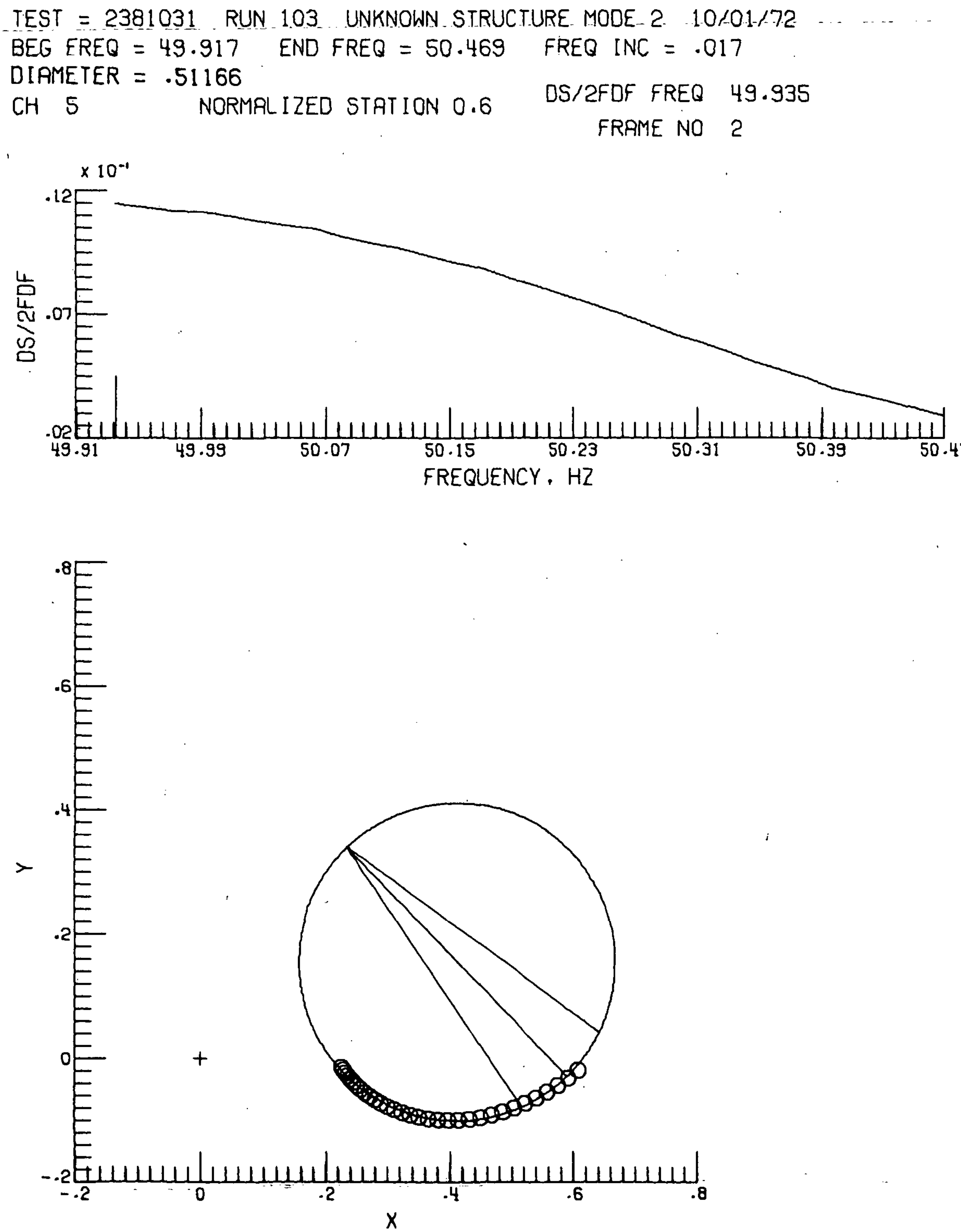
Figure A17.- Values of resonant frequency and damping for run 103, mode 1. (An asterisk indicates that the damping was calculated from an angle of 10°.)

TEST = 2381031 RUN 103 UNKNOWN STRUCTURE MODE 2 10/01/72
BEG FREQ = 49.917 END FREQ = 50.469 FREQ INC = .017
DIAMETER = 1.51682
CH 2 NORMALIZED STATION 0
DS/2FDF FREQ 50.171
FRAME NO 15



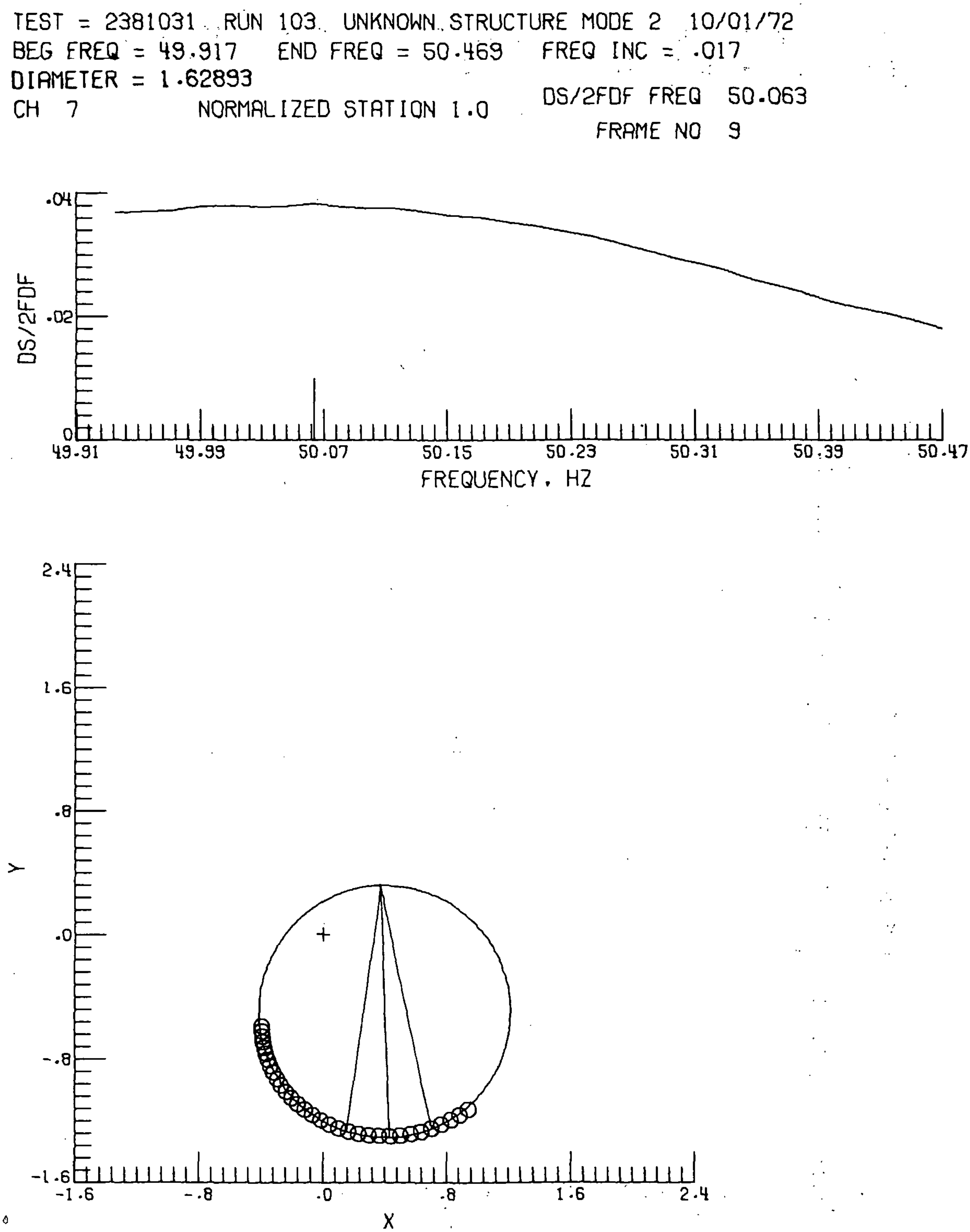
(a) Normalized station 0.

Figure A18.- Circle-fitted resonant response data plot and plot of $\Delta s/2f\Delta f$ against frequency for run 103, mode 2.



(b) Normalized station 0.6.

Figure A18.- Continued.



(c) Normalized station 1.0.

Figure A18.- Concluded.

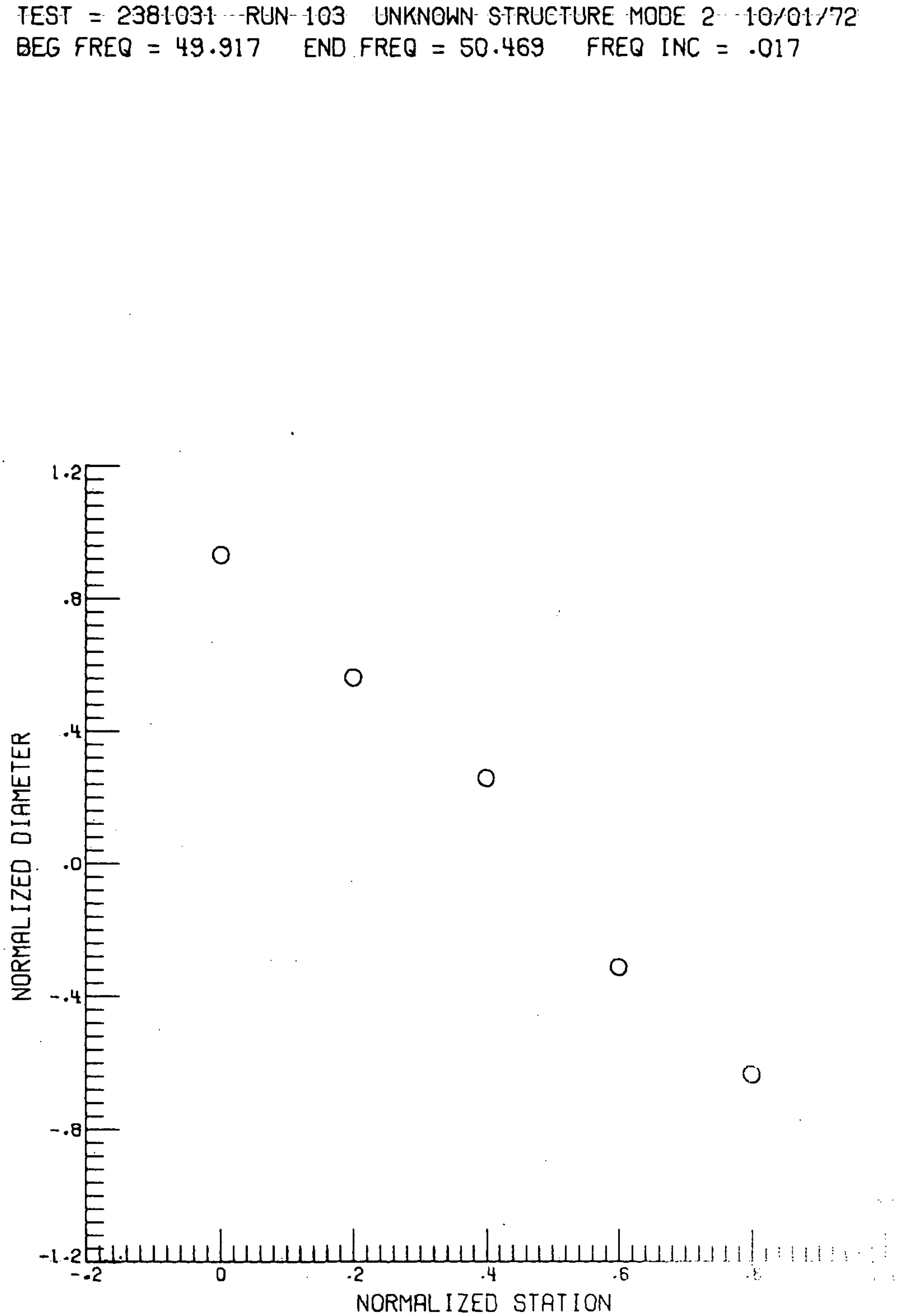
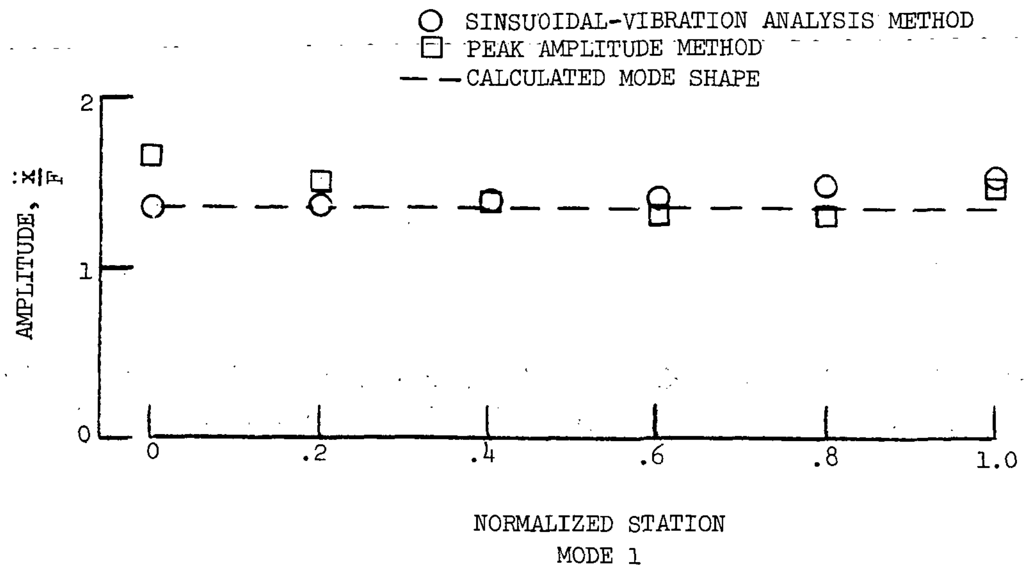


Figure A19.- Mode plot for run 103, mode 2.

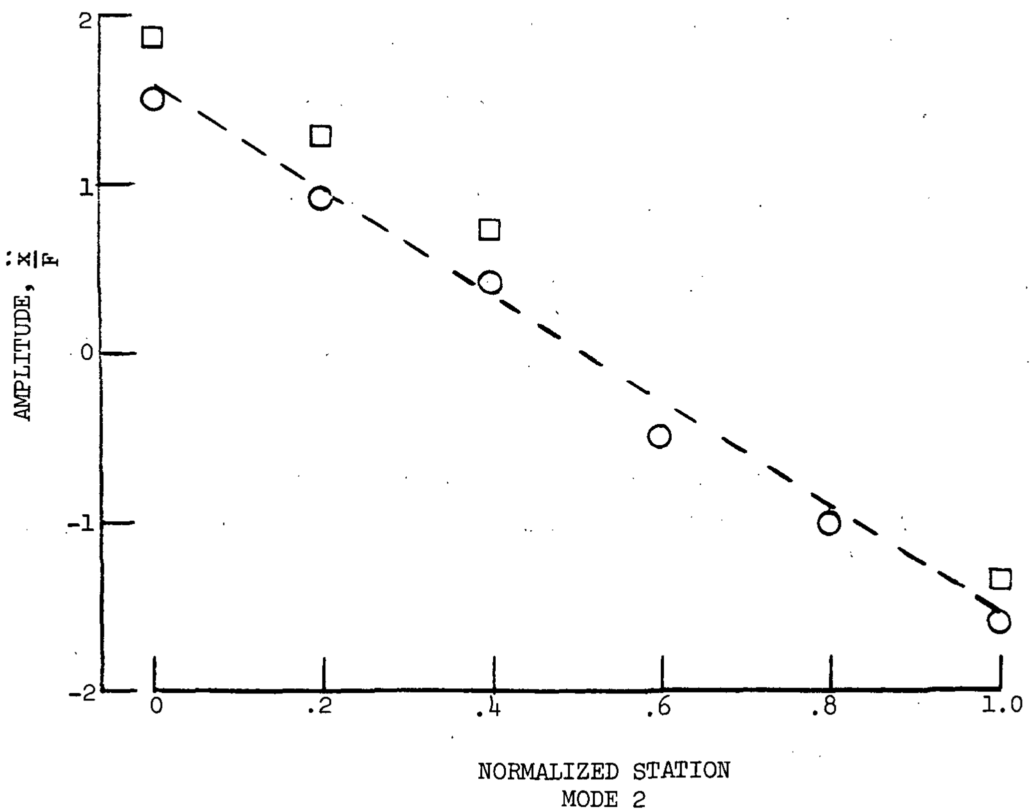
TEST = 2381031 RUN 103 UNKNOWN STRUCTURE MODE 2 10/01/72
BEG FREQ = 49.917 END FREQ = 50.469 FREQ INC = .017

CH	NOR STA	FREQUENCY	DIAMETER	DAMPING
2	0.000	50.171	1.51682	.01788
3	.200	50.206	.91441	.01811
4	.400	50.259	.41790	.02277
5	.600	49.935	-.51166	.01843*
6	.800	49.990	-1.03633	.01690*
7	1.000	50.063	-1.62893	.01709

Figure A20.- Values of resonant frequency and damping for run 103, mode 2. (An asterisk indicates that the damping was calculated from an angle of 10^{o}.)



(a) Mode 1 at 49.0 Hz.



(b) Mode 2 at 50.1 Hz.

Figure A21.- Modal amplitude plots.

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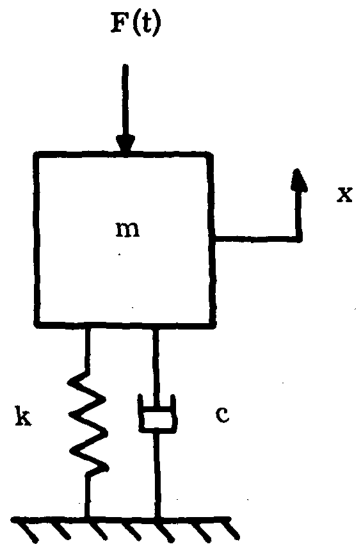
TABLE I. - ADTRAN FREQUENCY RANGE OF OPERATION

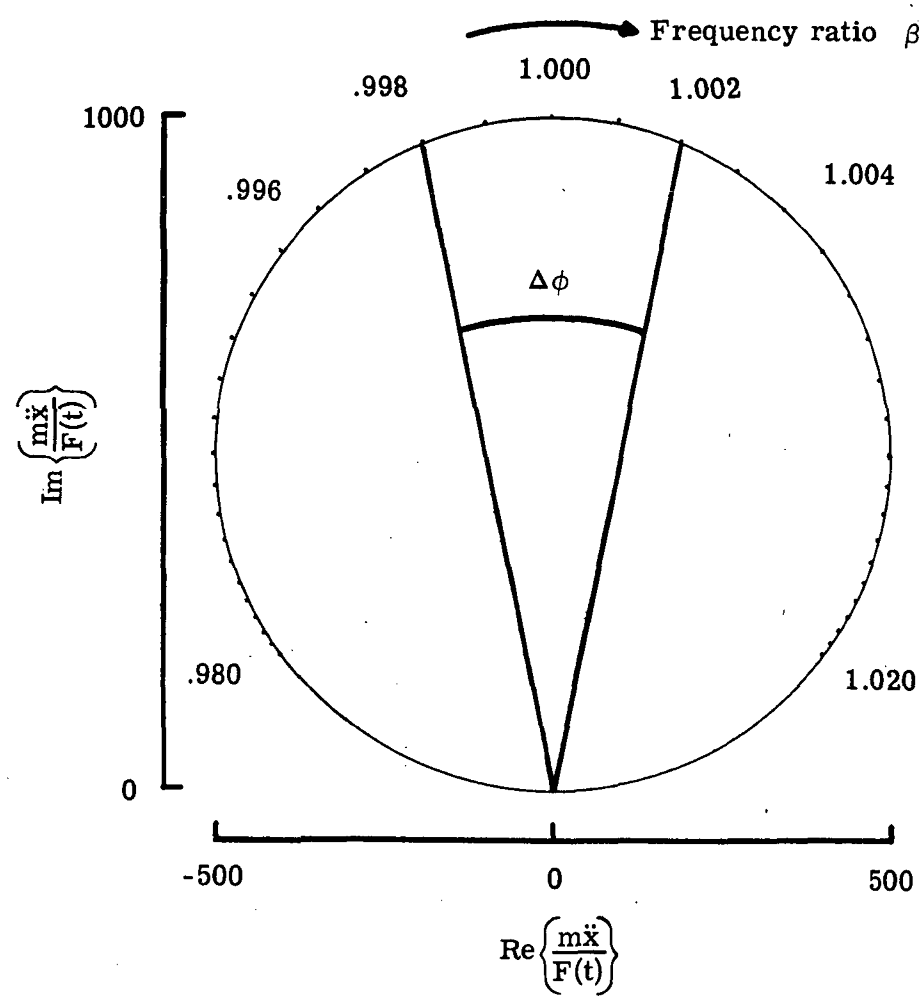
Frequency range, Hz	Frequency digitizing limitation		FM carrier center frequency, kHz	Tape recorder speed, ips*	Time code generator carrier frequency, Hz
	Record speed / Playback speed	Frequency range, Hz			
0.5 to 60	8	0.5 to 7.5	3.375	$1\frac{7}{8}$ or $3\frac{3}{4}$	1000
	2	2 to 30			
	1	4 to 60			
1 to 120	4	1 to 15	6.75	$3\frac{3}{4}$ or $7\frac{1}{2}$	1000
	1	4 to 60			
	1/2	8 to 120			
2 to 240	2	2 to 30	13.5	$7\frac{1}{2}$ or 15	1000
	1/2	8 to 120			
	1/4	16 to 240			
4 to 480	1	4 to 60	27	15 or 30	1000 or 10 000
	1/4	16 to 240			
	1/8	32 to 480			
8 to 960	1/2	8 to 120	54	30 or 60	10 000
	1/8	32 to 480			
	1/16	64 to 960			
16 to 2000	1/4	16 to 240	108	60 or 120	10 000
	1/16	64 to 960			
	1/32	128 to 1920			

*Depends on type of recorder.



$$F(t) = m\ddot{x} + c\dot{x} + kx$$

Figure 1.- Single-degree-of-freedom system.



$$\mu = \frac{114.5916(\Delta f)}{f_m \Delta\phi}$$

$$= \frac{114.5916(0.004)}{1(23)}$$

$$= 0.020$$

Figure 2. - Polar plot of the nondimensionalized inertance, and damping calculation from equation (6).

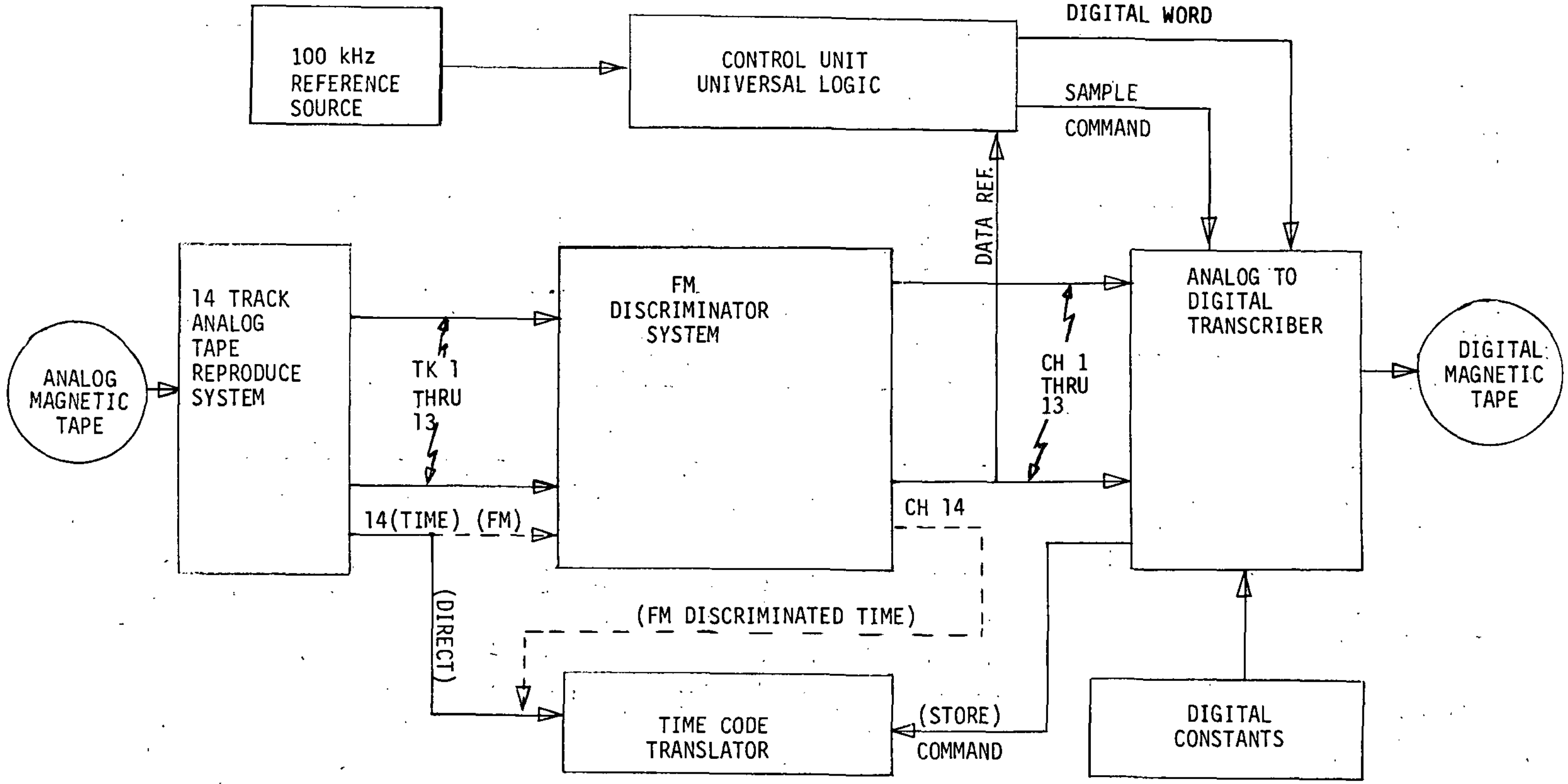


Figure 3.- Sinusoidal-vibration analysis data conversion system.

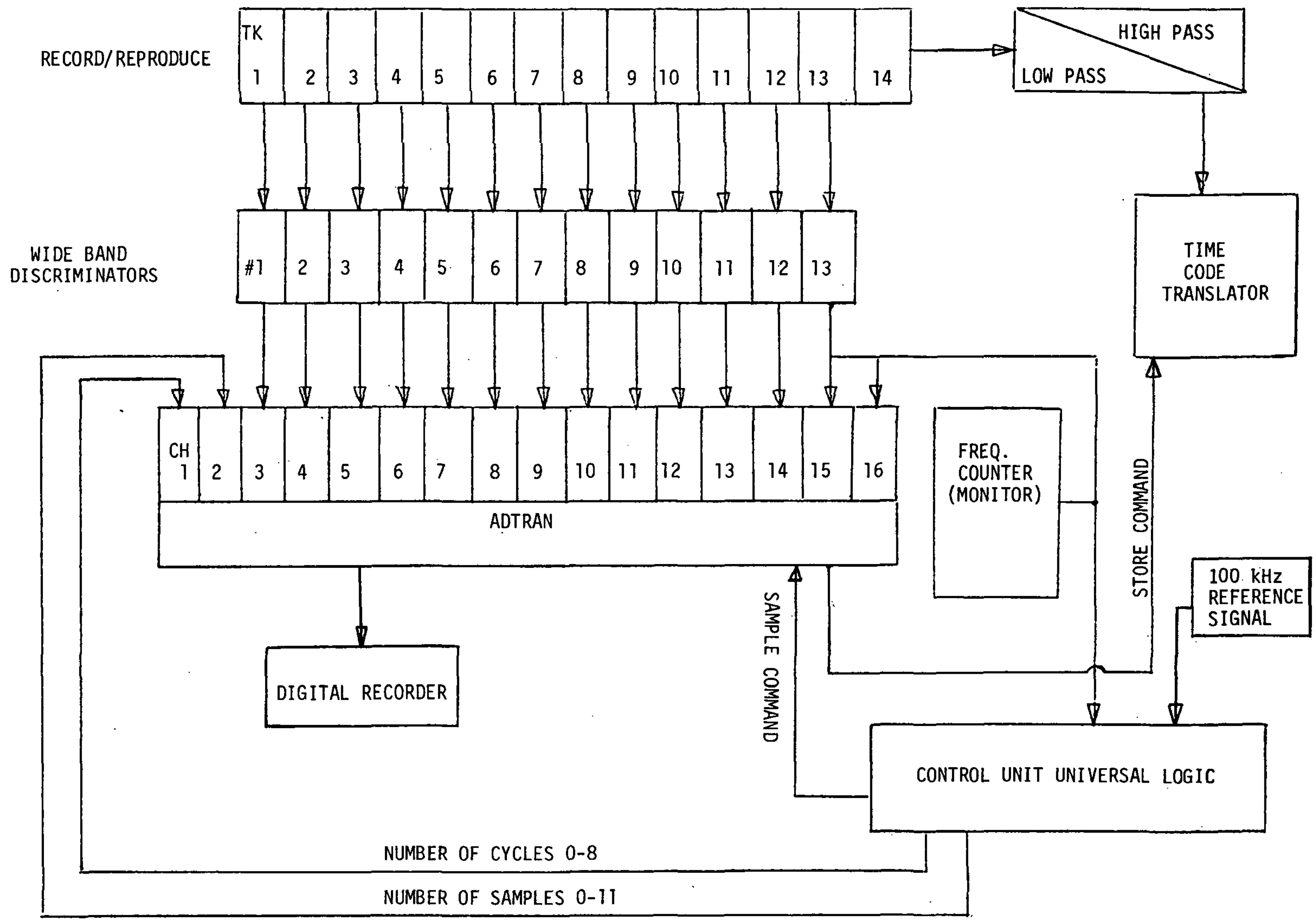


Figure 4. - Typical ADTRAN patching board for AC calibrations and data runs.

BCD 333 WORDS/RECORD 12 SAMPLES/CYCLE 3 CYCLES/RECORD

```
WORD   1      XX.XXXXX     SPEED
WORD   2       0000XXX     FILTER
WORD   3       000XXXX     SERIAL NUMBER
WORD   4      XXXXX.XX     LOAD LEVEL OF DISCRIMINATOR CALS
                           DC CALS (NOT PRESENTLY PROGRAMED)
WORD   5       XXXXXXX     TEST NUMBER   TEST XXX RUN XXX GROUP X
WORD   6       XXXXXXX     PERIOD
WORD   7     X/X/XXXXX     MODE, TYPE, BLOCK COUNTER
WORD   8    X/X/016/36     SYNC, SCALE, CHANNELS/FRAME, FRAMES/BLOCK
WORD   9       1XXXXXX     TIME (NOT USED)

WORD   10      0XXXXXX     TIME
WORD   11     ±XXXSXXX     CYCLE/SAMPLE          S IS SIGN DIGIT
WORD   12     ±XXXSXXX     CH 1/CH 2            9 +
WORD   13     ±XXXSXXX     CH 3/CH 4            8 -
WORD   14     ±XXXSXXX     CH 5/CH 6             A      B
WORD   15     ±XXXSXXX     CH 7/CH 8            ± XXX  S XXX
WORD   16     ±XXXSXXX     CH 9/CH 10           SIGN OF WORD APPLIES TO B
WORD   17     ±XXXSXXX     CH 11/CH 12          SIGN S APPLIES TO A
WORD   18     ±XXXSXXX     DUMMYCH/DUMMYCH

WORD   19, 28, 37, 46, 55, 64, 73, 82, 91, 100, 109, 118, 127, 136,
WORD   145, 154, 163, 172, 181, 190, 199, 208, 217, 226, 235, 244,    TIME
WORD   253, 262, 271, 280, 289, 298, 307, 316

WORD   325     0XXXXXX     TIME
WORD   326    ±XXXSXXX     CYCLE/SAMPLE
WORD   327    ±XXXSXXX     CH 1/CH 2
WORD   328    ±XXXSXXX     CH 3/CH 4
WORD   329    ±XXXSXXX     CH 5/CH 6
WORD   330    ±XXXSXXX     CH 7/CH 8
WORD   331    ±XXXSXXX     CH 9/CH 10
WORD   332    ±XXXSXXX     CH 11/CH 12
WORD   333    ±XXXSXXX     DUMMYCH/DUMMYCH
```

TAPE RECORDER DIGIT USED FOR INSTRUMENT TYPE ON CALIBRATIONS

36 SETS OF TIME, CYCLE/SAMPLE, CH1/CH2, CH3/CH4, CH5/CH6, CH7/CH8, CH9/CH10, CH11/CH12, DUMMYCH/DUMMYCH PER RECORD

Figure 5.- Digitized tape format.

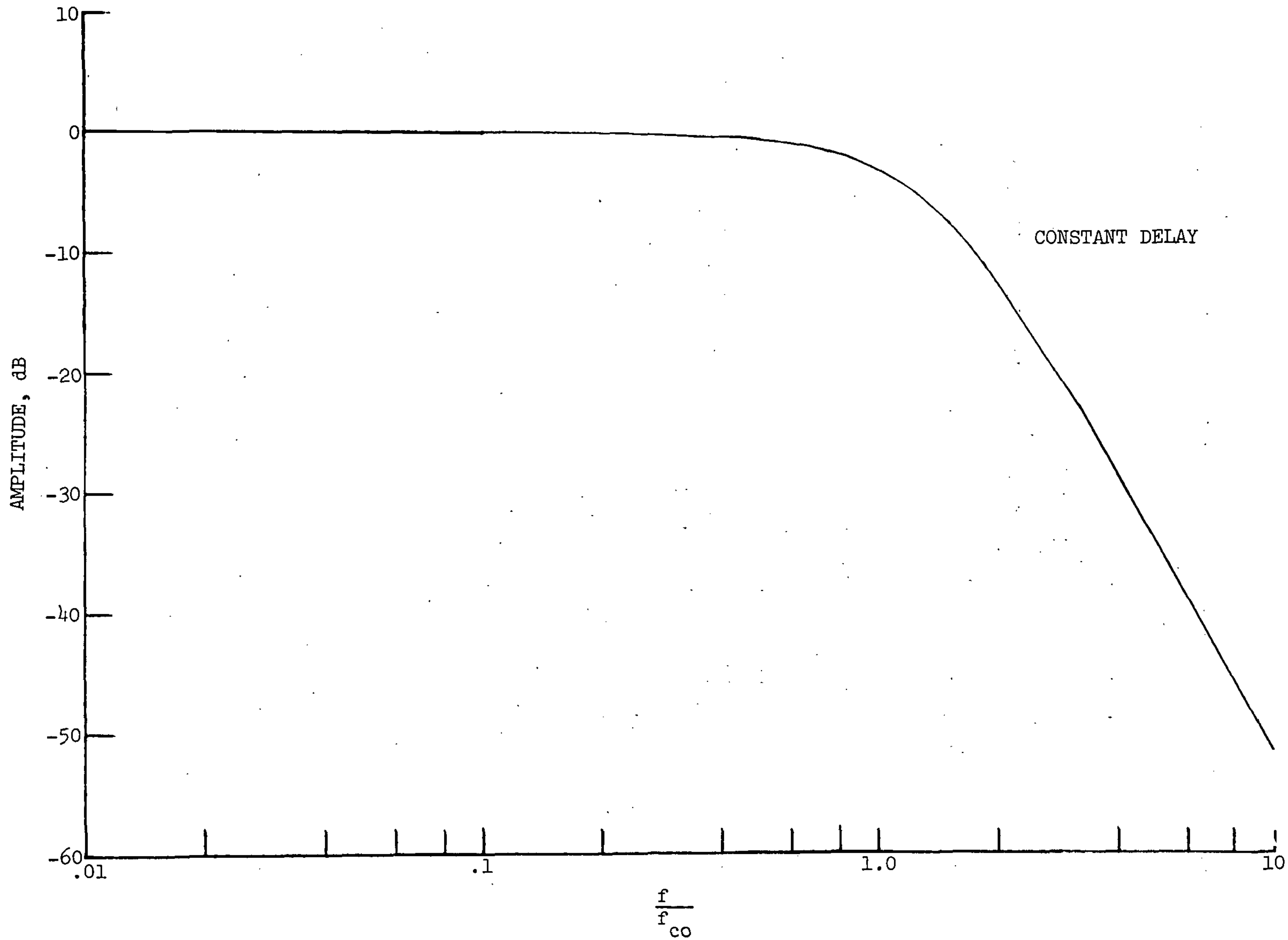


Figure 6.- Response characteristics of three-pole low-pass output filters.

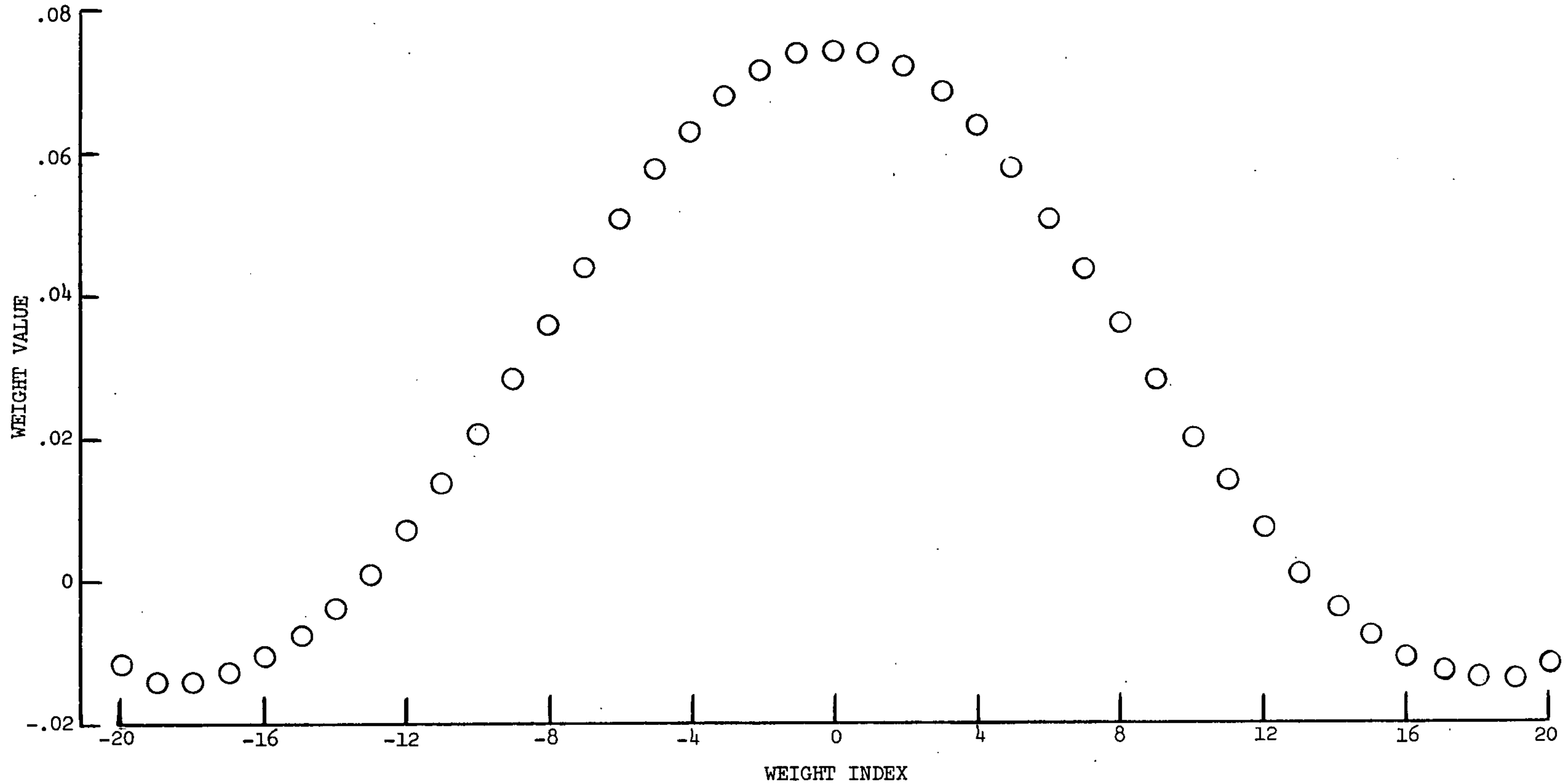


Figure 7.- Low-pass filter for 41 points with termination frequency of 0.5 Hz and cut-off frequency of 0.25 Hz.

BINARY 498 WORDS/RECORD BLOCKED 8 FRAMES/RECORD

TNOCH	WORD 01	NUMBER CHANNELS ON TAPE
SN	WORD 02	SERIAL NUMBER
TEST	WORD 03	TEST
RUN	WORD 04	RUN
GROUP	WORD 05	GROUP
FRAME	WORD 06	FRAME
TAPER	WORD 07	TAPE RECORDER NUMBER
R	WORD 08	SPEED RATIO
W	WORD 09	FREQUENCY CUT-OFF
DUMMY	WORD 10	NOT USED
FREQ	WORD 11, 72, 133, 194, 255, 316, 377, 438	FREQUENCY
CH1F	WORD 12, 73, 134, 195, 256, 317, 378, 439	CH 1 AMPLITUDE
CH1PA	WORD 13, 74, 135, 196, 257, 318, 379, 440	CH 1 PHASE ANGLE
CH1NF	WORD 14, 75, 136, 197, 258, 319, 380, 441	CH 1 NORMALIZED AMPLITUDE
CH1NPA	WORD 15, 76, 137, 198, 259, 320, 381, 442	CH 1 REFERENCED PHASE ANGLE
CH1D	WORD 16, 77, 138, 199, 260, 321, 382, 443	CH 1 ΔS/2FΔF
CH2F	WORD 17, 78, 139, 200, 261, 322, 383, 444	CH 2 AMPLITUDE
CH2PA	WORD 18, 79, 140, 201, 262, 323, 384, 445	CH 2 PHASE ANGLE
CH2NF	WORD 19, 80, 141, 202, 263, 324, 385, 446	CH 2 NORMALIZED AMPLITUDE
CH2NPA	WORD 20, 81, 142, 203, 264, 325, 386, 447	CH 2 REFERENCED PHASE ANGLE
CH2D	WORD 21, 82, 143, 204, 265, 326, 387, 448	CH 2 ΔS/2FΔF
CH3F	WORD 22, 83, 144, 205, 266, 327, 388, 449	CH 3 AMPLITUDE
CH3PA	WORD 23, 84, 145, 206, 267, 328, 389, 450	CH 3 PHASE ANGLE
CH3NF	WORD 24, 85, 146, 207, 268, 329, 390, 451	CH 3 NORMALIZED AMPLITUDE
CH3NPA	WORD 25, 86, 147, 208, 269, 330, 391, 452	CH 3 REFERENCED PHASE ANGLE
CH3D	WORD 26, 87, 148, 209, 270, 331, 392, 453	CH 3 ΔS/2FΔF
CH12F	WORD 67, 128, 189, 250, 311, 372, 433, 494	CH12 AMPLITUDE
CH12PA	WORD 68, 129, 190, 251, 312, 373, 434, 495	CH12 PHASE ANGLE
CH12NF	WORD 69, 130, 191, 252, 313, 374, 435, 496	CH12 NORMALIZED AMPLITUDE
CH12NPA	WORD 70, 131, 192, 253, 314, 375, 436, 497	CH12 REFERENCED PHASE ANGLE
CH12D	WORD 71, 132, 193, 254, 315, 376, 437, 498	CH12 ΔS/2FΔF

Figure 8.- Computed data output tape format.

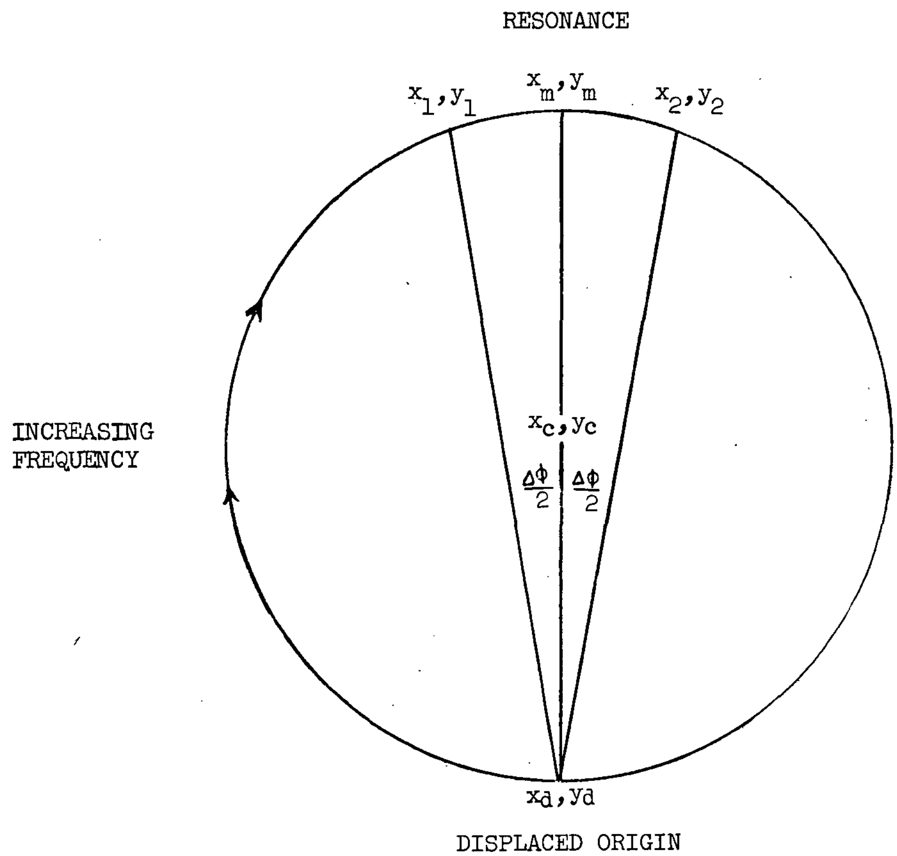


Figure 9.- Circle geometry.

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